Constraints on supersymmetric flavour models from $b \rightarrow s\gamma$

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ABSTRACT: We consider the effects of departures from minimal flavour violations (MFV) in the context of CMSSM-like theories. Second and third generation off-diagonal elements in the Yukawa, sfermion, and trilinear mass matrices are taken to be non-zero at the GUT scale. These are run down together with MSSM parameters to the electroweak scale. We apply constraints from fermion masses and CKM matrix elements to limit the range of the new free parameters of the model. We determine the effect of the departure from MFV on the branching ratio of $b \rightarrow s \gamma$. We find that only when the expansion parameter in the down-squark sector is relatively large there is a noticeable effect, which tends to relax the lower limit from $b \rightarrow s \gamma$ on the universal gaugino mass. We also find that the expansion parameter associated with the slepton sector needs to be smaller than the corresponding parameter in the down-squark sector in order to be compliant with the bound imposed by the branching ratio of $\tau \rightarrow \mu\gamma$.

KEYWORDS: Supersymmetry Phenomenology, B-Physics, Quark Masses and SM Parameters.

Contents

1. Introduction

Flavour symmetries (FS), [1]–[3], are constructed using experimental data from fermion masses and mixings, with or without imposing a supersymmetric framework. In theories with supersymmetric FS, there is no general principle which forbids off-diagonal terms in the soft squared mass or the trilinear mass matrices at the electroweak (EW) scale. Indeed supersymmetric FS are constructed so as to be independent of the specific mechanism for breaking supersymmetry. In addition, it is well known that in the minimal supersymmetric

standard model (MSSM), dangerous supersymmetric contributions to flavour changing processes can be avoided assuming flavour diagonal sfermion soft squared mass and trilinear mass matrices at the EW scale. However, there is no intrinsic reason why off-diagonal terms should be rotated away using the same unitary matrices which diagonalize the Yukawa matrices. In what follows, FS will always refer to supersymmetric flavour symmetries.

A supersymmetric theory can be considered to be minimal flavour violating (MFV) if flavour transitions occur only in the charged-current sector and such that can be entirely determined by the CKM angles [4]. In this sense, it is natural to assume that FS can be postulated assuming these MFV conditions. If FS are embedded in a supersymmetric grand unified theory (GUT) [1, 2], the Yukawa matrices have a well defined structure which contains off-diagonal terms at M_{GUT} and which evolve to the EW scale, reproducing the observed fermion masses and mixings. However, every possible assumption for the supersymmetric flavour structure at M_{GUT} can not be compatible with this version of MFV. Clearly, the running of general sfermion mass squared and trilinear mass matrices generate angles that cannot be absorbed by the CKM elements. As a consequence, it is often assumed that one can impose vanishing off-diagonal terms at the GUT scale in the soft squared mass and trilinear mass matrices [5]. In addition, when the trilinear terms are diagonal, the off-diagonal terms in the squark mass matrices generated radiatively at the EW scale can be rotated to the MFV form defined in [6] with the same unitary matrices diagonalizing the Yukawa matrices. In FS with supergravity [2, 7, 8], the scalar fields breaking such symmetries (flavons) generate off-diagonal terms at the GUT scale in the supersymmetric sector that are correlated to those of the fermion sector in a model dependent way. The available model dependent freedom allows one to construct safe theories at the EW scale.

Rather than determining the possible structure of the Yukawa matrices at the GUT scale (which is the basis for constructing FS) using only the fermion masses and mixings of the SM at the EW scale and hoping that the FCNC bounds can be satisfied, here we propose a bottom up approach for determining the parameters, both in the fermion sector and in the sfermion sector, of possible flavour symmetries taking into account (i) fermion masses with their corresponding supersymmetric corrections and mixings at the EW scale and (ii) measurements or bounds from FCNC experiments.

We will begin by first discussing the flavour violating processes that we will use in our subsequent analysis in section 2. We will focus primarily on the branching ratio for $b \to s\gamma$, given the current improvement in measuring its experimental value [9, 10]. We discuss its computation in models without MFV in section 2 as well. In section 3, we briefly discuss general features of the fermion mass matrices and the importance of considering their supersymmetric corrections in order to properly determine their profile at the GUT scale. We also point out that these theories naturally go beyond the MVF conditions at the EW scale.

In section 4, we employ conditions based on supergravity and FS to determine the form of the soft squared mass and trilinear mass matrices that we consider. The approach we take here makes use of an underlying supergravity theory with an effective Kähler potential from which one can construct the potential and through minimization, calculate the soft squared masses and trilinear couplings. It turns out that in supersymmetric theories with non-Abelian FS, the Kähler potential [2] has fewer model dependent parameters than other

sectors of the theory and hence can be effectively used to impose the size of the off-diagonal terms of soft squared mass and trilinear mass matrices at $M_{\rm GUT}$. Using the powerful tools of supergravity, one can then determine the structure in the supersymmetric sector at $M_{\rm GUT}$ and test its consequences at the EW scale. The form of the Yukawa matrices and soft terms are developed in section 5.

In section 6 we present the results of the analysis. We first focus on explaining the impact of considering non-diagonal Yukawa matrices at the GUT scale on the running of off-diagonal soft squared masses and trilinear couplings. Even when off-diagonal terms are taken to be zero at the GUT scale, due to the dependence of the beta functions of these terms, they will not be zero at the EW scale, but we show that they are well below current experimental bounds. We then determine the profile of the Yukawa matrices at the GUT scale when considering their supersymmetric corrections at the EW scale. We do not include GUT-specific effects and assume that all GUT-scale physics has decoupled below the GUT scale.

We then go on to determine the sensitivity of BR(b → s γ) on the parameters used to define the Yukawa couplings at the GUT scale. Although FCNC bounds serve to restrict the parameters determining the Yukawa matrices, soft squared matrices and trilinear couplings we find a region where the contribution to $b \to s\gamma$ is enhanced with respect to the constrained minimal supersymmetric standard model (CMSSM) with MFV and brings its value closer to the experimental value, thus relaxing the lower bound imposed on the universal gaugino mass at large $\tan\beta$. We show that this is possible while at the same time keeping flavour violating parameters in the lepton sector below the bounds imposed by BR($\tau \to \mu\gamma$). Finally, we mention the sensitivity of the determination of the MSSM spectra on these new parameters.

2. Supersymmetric flavour and CP violation problems in FS

It is well known that the supersymmetric soft Lagrangian, $\mathcal{L}_{\rm soft}^{\rm MSSM}$

$$
\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} = &-\frac{1}{2}\left(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + M_3\tilde{g}\tilde{g}\right) + h.c. \\
&-\left(\tilde{\bar{Q}}a_u H_u \tilde{u} + \tilde{\bar{Q}}a_d H_d \tilde{d} + \tilde{\bar{L}}a_e H_d \tilde{e}\right) + h.c. \\
&-\tilde{Q}M_{\tilde{Q}}^2\tilde{Q}^\dagger - \tilde{L}M_{\tilde{L}}^2\tilde{L}^\dagger - \tilde{u}M_{\tilde{u}}^2\tilde{u}^\dagger - \tilde{d}M_{\tilde{d}}^2\tilde{d}^\dagger - \tilde{e}M_{\tilde{e}}^2\tilde{e}^\dagger \\
&-M_{H_u}^2 H_u^* H_u - M_{H_d}^2 H_d^* H_d - (B\mu H_u H_d + c.c.)
\end{aligned}
\tag{2.1}
$$

introduces many new parameters that are not present in the SM. There are 106 mass terms, phases and mixings in the Lagrangian of the minimal version of the supersymmetric SM (MSSM), which cannot be rotated away by redefining the phases and flavour basis for the quark and lepton supermultiplets, and which have no counterpart in the SM.

However, most of the parameters involve flavour mixing or CP violation of the type which is severely restricted by experimental data. In the slepton sector, the most severe constraint is found when $M_{\tilde{e}}^2$ is not diagonal leading to the decay $\mu \to e\,\gamma$ which could take place, for example, via a one-loop diagram involving a virtual bino and a wino. The

current bound on BR($\mu \to e\gamma$) $\lesssim 10^{-11}$ [12] constrains all of the parameters involved in the decay and in particular restricts specific terms in the squared masses of the slepton sector. The current bound on BR($\tau \to \mu\gamma$) ($\lesssim 7 \times 10^{-8}$) [12] can also impose important constraints in the slepton sector.

Another example comes from $K^0 \leftrightarrow \overline{K}^0$ mixing where the effective Hamiltonian gets an important contribution from $M^2_{\tilde{d}\,12}$ via a box diagram involving gluinos. Winos and binos can also mediate these kind of diagrams. If the squark and gaugino masses are order $1\,\text{TeV}$ or less, it is possible to put bounds on the parameters Δm_K and ϵ_K appearing in the neutral kaon system effective Hamiltonian which restricts the amount of down-strange squark mixing and CP violating complex phases that one can tolerate among the soft parameters. Constraints related to the second and third generations come from the $D^o, \overline{D}^o$ and $B^o, \overline{B}^o$ neutral meson systems, and the decay $b \to s\gamma$. After the Higgs fields get vevs, the a_u, a_d and a_e matrices defined in eq. (2.1) contribute to off-diagonal squark and slepton mass squared terms and hence their form is also constrained by FCNC limits. There are other significant constraints on CP-violating phases from limits on the electric dipole moments of the neutron and electron and Hg [13].

All dangerous FCNC and CP violation effects in the MSSM can be avoided if one assumes that in the soft MSSM Lagrangian, eq. (2.1), all the mass matrices are diagonal. If the sfermion mass matrices are diagonal,

$$M^2_{\tilde{Q}} = m^2_{\tilde{Q}}\mathbf{1}, \ M^2_{\tilde{u}} = m^2_{\tilde{u}}\mathbf{1}, \ M^2_{\tilde{d}} = m^2_{\tilde{d}}\mathbf{1}, \ M^2_{\tilde{L}} = m^2_{\tilde{L}}\mathbf{1}, \ M^2_{\tilde{e}} = m^2_{\tilde{e}}\mathbf{1} \qquad (2.2)$$

then all squark and slepton mixings vanish. Squarks and sleptons with the same electroweak quantum numbers are degenerate in mass and can be rotated into each other. There are also supersymmetric contributions to FCNC modulo mixing due to a_u, a_d and a_e. Hence a further assumption usually implemented is that the trilinear terms are proportional to the Yukawa matrices

$$a_u = A_u Y_u, \quad a_d = A_d Y_d, \quad a_e = A_e Y_e, \qquad (2.3)$$

which ensures that only the squarks and sleptons of the third family can have large scalar cubic couplings.

Family symmetries attempt to reproduce mixings and masses in the quark and lepton sector usually by breaking the family symmetry spontaneously with scalar fields, *flavons*, and coupling different powers of them to quarks and leptons to reproduce the appropriate size of Yukawa couplings and hence explain the different hierarchies in fermion masses. However once these FS are implemented, if the theory is supersymmetric then not only will fermion mass terms be generated but sfermion mass terms as well and these will induce FCNC and CP violating effects. These effects then can be used to test different family symmetries. Unfortunately, unless the mechanism by which supersymmetry is broken is known, one is forced to make more assumptions, as in the case of the constrained version of the MSSM (CMSSM) discussed below.

In this paper, we will focus our attention to the constraints imposed by the rare decays of $b \to s \ \gamma$ on departures from MFV as the parameters that we consider in this analysis

are more sensitive to this process than to other observables such as $\mathrm{BR}(B_s \to \mu^+\mu^-)$ and ΔM_{B_s}. As a consequence, we will need to consider a generic method for computing $\mathrm{BR}(b \to s\,\gamma)$. QCD LO effects have been computed for the running of the chromo-magnetic Wilson coefficients of the effective Hamiltonian describing the decay $\mathrm{BR}(b \to s\,\gamma)$, from the EW scale to the decay scale, μ_b [14]. These coefficients arise as a result of the virtual exchange of gluinos, neutralinos, charged Higgs bosons and charginos.

The MFV QCD NLO contributions have been computed for the exchanges of charged Higgs boson and charginos in [4, 15 – 17]. In [18], the MFV MSSM NLO QCD contributions have been computed. These contributions complete the NLO calculation if the MFV condition is imposed at the electroweak scale. In addition, some beyond leading order effects (BLO), when the MFV is not assumed, have been calculated in [19].

Thus in a general framework, the only complete calculations of $\mathrm{BR}(b \to s\,\gamma)$ are available at LO. However, the present value of $\mathrm{BR}(b \to s\,\gamma)$ in the SM at NNLO is calculated [11]

$$\mathrm{BR}(b \to s\,\gamma) = (3.15 \pm 0.23) \times 10^{-4} \tag{2.4}$$

and the present experimental value estimated by the Heavy Flavour Average Group (HFAG) is [9, 10]

$$\mathrm{BR}(b \to s\,\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10}) \times 10^{-4}. \tag{2.5}$$

Here, we will not need to consider general departures from MFV at the EW scale. Instead, we are restricting the soft terms at the GUT scale. The expansion parameter of the u sector, ϵ_u, is taken to be much smaller than the corresponding parameter in the down sector, ϵ_d. In addition, we start with the Yukawa matrix, Y_u, diagonal at the GUT scale, and the departure from MFV will be sensitive mainly to gluinos. We provide the procedure that we use to obtain $\mathrm{BR}(b \to s\,\gamma)$ in the framework described below in appendix A.

3. Fermion masses and MFV conditions

When flavour symmetries are constructed, the quark sector is usually fixed by precision fits of the CKM elements. The lepton sector is similarly determined by the UMNS elements although only two mixing angles out of the three mixing angles and three phases (one CKM-like and two Majorana) are known. Thus, the determination of the lepton mixing matrix is not precise as in the case of the quark sector. Hence it is more common to construct flavour symmetries by fixing elements of Yukawa matrices in the quark sector and then adjusting parameters in the lepton sector by relating these to the quark sector and of course using the available information from leptons. Since only the left handed states enter into the electroweak mixings, only the Yukawa couplings along and above (or below, depending on the convention) the diagonal can be determined, so it is necessary to make an assumption of the Yukawa couplings below (or above) the diagonal in the Yukawa matrix. It is natural then to classify these choices as symmetric or non-symmetric Yukawa matrices. For the non-symmetric matrices, however there is less precision in determining

the elements because the number of parameters is larger than in the symmetric case. The form of these matrices for the symmetric case is [25]:

$$Y_{d\ sym} \propto \begin{bmatrix} \le \epsilon_d^4 & \epsilon_d^3 & \le \epsilon_d^3 \\ \epsilon_d^3 & \le \epsilon_d^2 & \epsilon_d^2 \\ \le \epsilon_d^3 & \epsilon_d^2 & 1 \end{bmatrix}, \quad Y_{u\ sym} \propto \begin{bmatrix} \le \epsilon_u^4 & \epsilon_u^3 & \le \epsilon_u^3 \\ \epsilon_u^3 & \le \epsilon_u^2 & \epsilon_u^2 \\ \le \epsilon_u^3 & \epsilon_u^2 & 1 \end{bmatrix}, \qquad (3.1)$$

while for the non-symmetric [25] case we have:

$$Y_d \propto \begin{bmatrix} \le \epsilon_d^4 & \epsilon_d^3 & \le \epsilon_d^3 \\ \le \epsilon_d^3 & \le \epsilon_d^2 & \epsilon_d^2 \\ \le \epsilon_d & \le 1 & 1 \end{bmatrix}, \quad Y_u \propto \begin{bmatrix} \le \epsilon_u^4 & \epsilon_u^3 & \le \epsilon_u^3 \\ \le \epsilon_u^3 & \le \epsilon_u^2 & \epsilon_u^2 \\ \le \epsilon_u & \le 1 & 1 \end{bmatrix}, \qquad (3.2)$$

with $\epsilon_d = O(\lambda_c)$ and $\epsilon_u = O(\lambda_c^2)$, where $\lambda_c \approx 0.23$ is the Wolfenstein parameter $V_{\mathrm{us}} = \lambda_c$. The coefficients of Yukawa elements are taken to be $O(1)$. In general, we are free to independently choose either Y or Y_{sym} for the up-type and down-type sectors. As the non-symmetric matrices have more parameters, we will restrict our attention here to the symmetric cases which will be the most restrictive.

The extension of FS to the supersymmetric sector is somewhat arbitrary, independently of the many complications inherent in the MSSM. The problem rests in the lack of an underlying theory which relates the parameters of a FS to the supersymmetric parameters. From a bottom up approach we can tackle this problem by using the structure of the running of the soft parameters in the MSSM.

The global symmetry of the gauge sector of the SM is given by $\mathrm{U}(3)^5 = \mathrm{SU}(3)_Q \times \mathrm{SU}(3)_U \times \mathrm{SU}(3)_D \times \ldots$, and is broken only by the Yukawa couplings

$$Y_d \to \overline{3}_Q \times 3_d, \quad Y_u \to \overline{3}_Q \times 3_u, \quad Y_e \to \overline{3}_L \times 3_e. \qquad (3.3)$$

The MFV hypothesis [4] basically constrains this breaking to the fermion sector and imposes eq. (2.2) and eq. (2.3) at some low scale to ensure small FCNCs.

Given the dependence of the beta functions of the MSSM soft terms [26] on $Y_f^\dagger Y_f$ and $Y_f Y_f^\dagger$, their running will alter the conditions eq. (2.2) and eq. (2.3) and hence the MFV scale is set at the EW scale. Thus, within the MSSM, the MFV hypothesis implies a strong restriction on the scale at which FCNCs are evaluated (e.g. μ_b):

$$\tilde{Q} M_{\tilde{Q}}^2 \tilde{Q}^\dagger \quad \propto \tilde{Q} \left[\sum x_n \left(Y_u Y_u^\dagger \right)^n \right] \tilde{Q}^\dagger \sim \tilde{Q} \left[x_o I + x_1 Y_u Y_u^\dagger \right] \tilde{Q}^\dagger \qquad (3.4)$$

As a consequence we obtain the same CKM factors for the soft masses as in the SM and only the flavour-independent magnitude of FCNC amplitudes are modified, for example:

$$A(b \to s\gamma) \propto [(V_{\mathrm{CKM}})_{\mathrm{ts}}^* (V_{\mathrm{CKM}})_{\mathrm{tb}}], \quad \Delta M_{B_{d,s}} \propto [(V_{\mathrm{CKM}})_{\mathrm{tb}} (V_{\mathrm{CKM}})_{td,s}]^2 \qquad (3.5)$$

Of course, the motivation for the MFV conditions is the strong suppression of BSM effects in flavour parameters (e.g. SM CKM fits). However, one must check carefully that the form of $\tilde{Q} M_{\tilde{Q}}^2 \tilde{Q}^\dagger$ in fact does not violate that assumption in the sense that it could give a large

deviation from the MFV conditions. Once the running of Yukawa matrices is included this can never be the case.

If we put the MFV conditions at the GUT scale for arbitrary initial conditions, this can result into catastrophic FCNCs at the EW scale. Since the main goal of FS is to restrict the form of Yukawa matrices, Y_f, we then know the form of the terms

$$Y_f^\dagger Y_f, \quad Y_f Y_f^\dagger, \quad a_f^\dagger a_f \quad \text{and} \quad a_f a_f^\dagger \tag{3.6}$$

at the GUT scale. We can restrict then the initial conditions of off-diagonal terms of the soft parameters by the off-diagonal elements of the terms eq. (3.6). The specific way of correlating this can be obtained with a FS governed by a non-minimal supergravity theory.

The MFV conditions in supersymmetry have been extended to include terms such as $x_2 Y_d Y_d^\dagger + x_3 Y_d Y_d^\dagger Y_u Y_u^\dagger + x_4 Y_u Y_u^\dagger Y_d Y_d^\dagger$ in eq. (3.4) and a similar structure to other supersymmetric terms [6]. In this case, gluino contributions to FCNC can compete with those of chargino and charged Higgs contributions. We present examples below of such cases when A_0 is diagonal at the GUT scale.

4. Flavour symmetries with an underlying supergravity theory

4.1 Trilinear terms

The expansion parameters appearing in the Yukawa couplings, ϵ_d and ϵ_u, in flavour symmetries arise due to scalar fields, flavons, acquiring a vev, $\langle \theta \rangle$, and hence breaking the flavour symmetry at some scale M:

$$(Y_f)_{ij} \propto \left(\frac{\langle \theta \rangle}{M} \right)^{\alpha_{ij}^f}, \tag{4.1}$$

For example in SU(3) flavour symmetries, the Yukawa matrices have the form

$$Y_f/(Y_f)_{33} = \begin{pmatrix} 0 & c_{12}^f \epsilon_f^3 e^{i\varphi_{12}^f} & c_{13}^f \epsilon_f^3 e^{i\varphi_{13}^f} \\ c_{21}^f \epsilon_f^3 e^{i\varphi_{21}^f} & c_{22}^f \epsilon_f^2 e^{i\varphi_{22}^f} & c_{23}^f e^{i\varphi_{23}^f} \epsilon_f^2 \\ c_{31}^f \epsilon_f^3 e^{i\varphi_{31}^f} & c_{32}^f e^{i\varphi_{32}^f} \epsilon_f^2 & 1 \end{pmatrix}, \tag{4.2}$$

Note that we will be assuming a symmetric form for the Yukawa matrices so that $c_{ij} = c_{ji}$ and we drop the subscript *sym*. As noted earlier, there are only 3 physical CP violating phases and therefore there are relations between the phases φ_{ij}^f in eq. (4.2). This equation represents the most general parameterization of the Yukawa matrices for quarks and charged leptons with an SU(3) FS [2, 3]. In any supersymmetric FS, the flavon fields also couple to the sfermions through the trilinear terms

$$(a_f)_{ij} H_f Q_i q_j^c. \tag{4.3}$$

The generic form of the trilinear coupling matrices in models of supersymmetry breaking such as supergravity mediation, gauge mediation or anomaly mediation, is of the form

$$(a_f)_{ij} = (Y_f)_{ij} \left(A_0^f \right)_{ij}. \tag{4.4}$$

In minimal supergravity A_0 is a constant and hence the proportionally in eq. (2.3) is achieved. Once a family symmetry is considered [2], there are additional contributions to the trilinear couplings given by derivatives of the Yukawa couplings with respect to the flavon fields [2, 8], and the trilinear terms can then be written as

$$(a_f)_{ij} = (Y_f)_{ij} \left(\left(A_0^f \right)_{ij} + k_{ij}^f \right),$$

$$(4.5)$$

where the k_{ij}^f are the coefficients produced when taking the derivatives with respect to the flavon fields times the gravitino mass, $k_{ij}^f \propto \alpha_{ij}^f m_{3/2}$.

4.2 Soft squared masses

The soft squared mass terms in general scenarios of soft supersymmetry breaking remain diagonal, although the diagonal terms can be different from each other. However in theories with underlying supergravity and family symmetries, the off-diagonal terms involve powers of the flavon fields. Of course the diagonal terms are free of such contributions since the terms in the Lagrangian involving the soft squared masses, eq. (2.1), need to respect Hermiticity.

Once the flavour symmetry is specified, the Kähler potential can be trivially written as

$$K = \sum_\psi \psi^i \psi^{\dagger \bar{j}} K_{i\bar{j}}(\psi), \qquad \psi = u_R, \ d_R, \ e_R, \ \nu_R, \ Q_L, \ L_L, \qquad (4.6)$$

$$K_{i\bar{j}}(\psi) = \delta_{i\bar{j}} \left[c(\psi) + d(X_p, \psi) X^p X_p^\dagger \right] + \frac{\theta_i^a \theta_{a\bar{j}}^\dagger}{M^2(\theta_a)} \left[c(\theta_a, \psi) + d(\theta_a, X_p, \psi) X^p X_p^\dagger \right]. \ (4.7)$$

Here θ_k are the flavon fields and X_p are the fields breaking supersymmetry. The coefficients c and d are a-priori un-related coefficients that can be different for each different flavon and X field. In abelian flavour symmetries, they are different for each fermion species but in non-Abelian symmetries the coefficients of one species to another are often related. For example in SU(3) flavour models, [2, 3, 25], we have structures like

$$M^2 = m_{3/2}^2 \mathbf{1} - m_0^2 \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & O\left(\frac{\langle\theta\rangle^2}{M^2}\right) \\ 0 & O\left(\frac{\langle\theta\rangle^2}{M^2}\right) & r_3 \end{pmatrix},$$

$$(4.8)$$

where it is important to stress that due to the vacuum alignment of the flavon fields, [2, 3] there are no D terms contributing to the soft-squared masses, which often present dangerous contributions to FCNC [27]. In what follows, we adopt the following form for the soft-squared masses

$$M_{\tilde{f}}^2 = \begin{pmatrix} M_{\tilde{f}\,11}^2 & 0 & 0 \\ 0 & M_{\tilde{f}\,22}^2 & M_{\tilde{f}\,23}^2 \\ 0 & M_{\tilde{f}\,23}^{2\dagger} & M_{\tilde{f}\,33}^2 \end{pmatrix}, \quad f = Q, u, d, L, e.$$

$$(4.9)$$

5. Determination of the flavour structure at the GUT scale

5.1 General assumptions

Having specified the assumptions that we make based on an underlying supergravity structure, we next describe how these structures are realized at the electroweak scale and how we might implement experimental constraints on the structure at the GUT scale. In a complete theory of FS, the full superpotential and Kähler potential would be specified, including all the coefficients appearing in eq. (4.1), eq. (4.5) and eq. (4.7). One can then impose those initial conditions at the GUT scale and run those parameters down to the EW scale. Lacking a complete theory we can at best begin with the general form of the Yukawa matrices, eq. (3), and the supergravity assumptions described in the previous section at the GUT scale and apply the constraints from FCNC, mainly $BR(b \to s\gamma)$ to limit the possible values of the parameters of the flavour structure for Yukawa matrices and soft terms at the GUT scale. To this end, we implement an iterative procedure, like the one followed in analyses of the constrained MSSM (CMSSM) [28, 29].

In the CMSSM, gaugino masses are assumed to be unified at the GUT scale with value $m_{1/2}$, as are the soft scalar masses with value m_0 and trilinear couplings with value A_0. Typically, the flavour structure for the scalar masses and trilinear terms is assumed to be diagonal at both the GUT and EW scales. That is, the running of the off-diagonal terms is also neglected. With this set of boundary conditions at the GUT scale, we can use the radiative electroweak symmetry breaking conditions by specifying the ratio of the two Higgs vacuum expectation values, $\tan\beta$, and the mass, M_Z, to predict the values of the Higgs mixing mass parameter, μ and the bilinear coupling, B. The sign of μ remains free.

The CMSSM is subject to a number of phenomenological constraints. These include the LEP limits on the chargino mass: $m_{\chi^\pm} > 104\,\text{GeV}$ [30], on the selectron mass: $m_{\tilde{e}} > 99\,\text{GeV}$ [31] and on the Higgs mass: $m_h > 114\,\text{GeV}$ [32]. The former two constrain $m_{1/2}$ and m_0 directly via the sparticle masses, and the latter indirectly via the sensitivity of radiative corrections to the Higgs mass to the sparticle masses, principally $m_{\tilde{t},\tilde{b}}$. The Higgs limit imposes important constraints principally on $m_{1/2}$ particularly at low $\tan\beta$. Another constraint is the requirement that the branching ratio for $b \to s\gamma$ is consistent with the experimental measurements [9, 10]. These measurements agree at the 1-σ level with the Standard Model, and therefore provide bounds on MSSM particles [17], such as the chargino and charged Higgs masses, in particular. Typically, the $b \to s\gamma$ constraint is more important for $\mu < 0$, but it is also relevant for $\mu > 0$, particularly when $\tan\beta$ is large. The constraint imposed by measurements of $b \to s\gamma$ also excludes small values of $m_{1/2}$. To apply the constraint from $b \to s\gamma$ to the supersymmetric parameter space, we convolve the experimental result given in eq. (2.5) with the theoretical calculated result and uncertainty. We add linearly the uncertainty generated from the variation of the input matching scale to the SM error given in eq. (2.4). We also add linearly the systematic uncertainties in the experimental result given in eq. (2.5) which could allow central values of $b \to s\gamma$ significantly outside the range inferred from the statistical error in eq. (2.5). For example, in the CMSSM, the 95% CL lower limit to $m_{1/2}$ at high $\tan\beta$ occurs when the calculated central value of the branching ratio of $b \to s\gamma$ is as low as 1.9×10^{-4} [33]. Finally, there are

regions of the $(m_{1/2}, m_0)$ plane that are favoured by the BNL measurement [34] of $g_\mu - 2$ at the 2-σ level, corresponding to a deviation from the Standard Model calculation [35] using e^+e^- data.

If R-parity is conserved, the lightest supersymmetric particle becomes a cold dark matter candidate [36] and therefore the model becomes subject to cosmological constraints on the relic density of dark matter (usually the lightest neutralino) [29]. In the context of the ΛCDM model, the WMAP only results indicate [37]

$$\Omega_{\text{CDM}} h^2 = 0.1042^{+0.0081}_{-0.0080} \tag{5.1}$$

or a 2σ range of 0.0882 – 0.1204 for $\Omega_{\text{CDM}} h^2$. This range of densities forces one into relatively narrow regions in a $m_{1/2}, m_0$ plane when $\tan\beta$ and A_0 are fixed.

In this paper, we will restrict our attention to models with fixed $\tan\beta = 50$ with $\mu > 0$. This is done to enhance somewhat the importance of $b \to s\gamma$ constraints. We take $m_t = 171.4\,\text{GeV}$ and $m_b(m_b) = 4.25\,\text{GeV}$. In the CMSSM with $\tan\beta = 50$ and $A_0 = 0$, the Higgs mass constraint requires $m_{1/2} \gtrsim 350\,\text{GeV}$ and is similar to the constraint from $b \to s\gamma$ for this choice of $\tan\beta$ and A_0. At larger values of $m_{1/2}$, the value of m_0 is constrained to lie in a relatively narrow range to ensure that the co-annihilation of neutralinos and staus are effective in reducing the relic density to the WMAP range. At very large values of $m_{1/2}$, s-channel annihilations of neutralinos through heavy Higgs scalars and pseudo-scalars control the relic density and larger values of m_0 are allowed. The parameter choices used in the results presented in the next section have been made such that the the relic density is within the WMAP range in the CMSSM. We also note that to explain the $g_\mu - 2$ result at the 2-σ level, places an upper bound on $m_{1/2}$ of approximately $900\,\text{GeV}$. We have not considered here parameter choices with very large values of m_0 typically associated with the focus point as this part of the plane is not very sensitive to FC processes.

5.2 Form of the Yukawa matrices and soft terms

Our goal in this paper is to test the effect of including off-diagonal terms in the scalar and trilinear matrices. We will restrict our attention to the mixing between the second and third generations only (mixing with the first generation will be of higher order in the expansion parameters ϵ_u or ϵ_d as discussed in the previous section). At the EW scale, we set up off-diagonal entries of Y_d using the CKM mixings. We also assume the same mixings for the charged lepton sector. At the GUT scale we choose the following form of the Yukawa matrices, which contain 10 parameters:

$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{22}^u \epsilon_u^2 & 0 \\ 0 & 0 & c_{33}^u \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{22}^d \epsilon_d^2 & c_{23}^d \epsilon_d^2 \\ 0 & c_{23}^d \epsilon_d^2 & c_{33}^d \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{22}^e \epsilon_d^2 & c_{23}^e \epsilon_d^2 \\ 0 & c_{23}^e \epsilon_d^2 & c_{33}^e \end{pmatrix}. \tag{5.2}$$

In the up sector, we have the masses of the top and charm quarks to determine two of the three parameters in Y_u, thus we have just one free parameter which we choose to be ϵ_u. The d and e sectors are linked because we are choosing to generate the mixing in the lepton sector using the V_{CKM} matrix:

$$Y_d = V_{\text{CKM}}^* \hat{Y}_d V_{\text{CKM}}^\dagger, \quad Y_e = V_{\text{CKM}}^* \hat{Y}_e V_{\text{CKM}}^\dagger, \tag{5.3}$$

where $\hat{Y}_f$ are the diagonal matrices of the bare Yukawa couplings at the EW scale. In the e and d sectors combined, there are seven parameters. We can use the masses of μ, τ, and the masses of the strange and bottom quarks to eliminate four parameters. An additional parameter is eliminated by the V_{CKM} matrix, leaving us with two free parameters which we choose to be ϵ_d and c_{22}^d. Thus choosing

$$\epsilon_u, \quad \epsilon_d, \quad \text{and} \quad c_{22}^d \tag{5.4}$$

as free parameters at the GUT scale allows us to determine the remaining coefficients

$$c_{22}^u, \quad c_{33}^u, \quad c_{23}^d, \quad c_{33}^d, \quad c_{22}^e, \quad c_{23}^e, \quad \text{and} \quad c_{33}^e. \tag{5.5}$$

The fits are done at the appropriate low energy scale and run up and down iteratively to obtain a consistent solution given our set of input parameter values.

Although we are choosing ϵ_u and ϵ_d as free parameters, their range is restricted by requiring convergence of the running of the RGEs. Outside this range, Yukawa couplings tend to diverge while running down to the EW scale. This is reminiscent of the restriction to $\tan\beta$ in the CMSSM. The allowed range can be estimated from the running of the mass ratios m_s/m_b and m_c/m_t, which are directly correlated to the values ϵ_d and ϵ_u respectively by eq. (38). These ratios increase from the EW to the GUT scale in the MSSM. We use an analytic form [38] for the ratios of Yukawa couplings derived from one-loop RGEs for quark mixings and solved in the presence of the heaviest Yukawa generation. In this approach the evolution parameter, χ is simply related to the evolution of the mass ratios m_s/m_b and m_c/m_t by

$$\chi = \left(\frac{M_G}{M_Z}\right)^{-y_t^2/16/\pi^2} \approx 0.8, \quad \frac{(m_s/m_b)(M_G)}{(m_s/m_b)(M_Z)} = \chi, \quad \frac{(m_c/m_t)(M_G)}{(m_c/m_t)(M_Z)} = \chi^3. \tag{5.6}$$

Then at M_{GUT}, $\epsilon_d \approx \sqrt{m_s/m_b} \approx 0.1$ and $\epsilon_u \approx \sqrt{m_c/m_t} \approx 0.05$ and hence the range for ϵ_d and ϵ_u should be restricted to be close to 0.1 and 0.05 respectively.

With the above parameterization of Yukawa matrices then we can parameterize the soft masses as follows:

$$M_{\tilde{Q}}^2 = M_{\tilde{u}_R}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & x_{22}^u & x_{23}^u \epsilon_{\text{Su}}^2 \\ 0 & x_{23}^u \epsilon_{\text{Su}}^2 & x_{33}^u \end{pmatrix} m_0^2, \qquad M_{\tilde{d}_R}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & x_{22}^d & x_{23}^d \epsilon_{\text{Sd}}^2 \\ 0 & x_{23}^d \epsilon_{\text{Sd}}^2 & x_{33}^d \end{pmatrix} m_0^2$$

$$M_{\tilde{L}}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & x_{22}^\nu & x_{23}^\nu \epsilon_{\text{Su}}^2 \\ 0 & x_{23}^\nu \epsilon_{\text{Su}}^2 & x_{33}^\nu \end{pmatrix} m_0^2, \qquad M_{\tilde{e}_R}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & x_{22}^e & x_{23}^e \epsilon_{\text{Se}}^2 \\ 0 & x_{23}^e \epsilon_{\text{Se}}^2 & x_{33}^e \end{pmatrix} m_0^2$$

$$a_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & z_{22}^u \epsilon_{\text{Su}}^2 & 0 \\ 0 & 0 & z_{33}^u \end{pmatrix} A_0, \qquad a_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & z_{22}^d \epsilon_{\text{Sd}}^2 & z_{23}^d \epsilon_{\text{Sd}}^2 \\ 0 & z_{23}^d \epsilon_{\text{Sd}}^2 & z_{33}^d \end{pmatrix} A_0,$$

$$a_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & z_{22}^e \epsilon_{\text{Se}}^2 & z_{23}^e \epsilon_{\text{Se}}^2 \\ 0 & z_{23}^e \epsilon_{\text{Se}}^2 & z_{33}^e \end{pmatrix} A_0 \tag{5.7}$$

In the spirit of the CMSSM, we can simply choose all of the $x_{ij}^f = 1$, $\epsilon_{\text{S}f} = \epsilon_f$, and set the trilinear couplings to be aligned with the Yukawa matrices, such that $z_{ij}^f = c_{ij}^f$. We will make this simplification in sections 6.1 - 6.3.

Later, in section 6.4, we will use the generality obtained when $\epsilon_{\text{Sd}}, \epsilon_{\text{Se}}$, and ϵ_{Su} are assumed to be unrelated to ϵ_d and ϵ_u to set constraints on these parameters from $b \to s\, \gamma$. We can test departures from the CMSSM by effectively taking coefficients x_{ij}^f different from unity, but that is clearly beyond the scope of the present study. The matrices are then simplified to:

$$M_{\tilde{Q}}^2 = M_{\tilde{u}_R}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_{\text{Su}}^2 \\ 0 & \epsilon_{\text{Su}}^2 & 1 \end{pmatrix} m_0^2, \qquad M_{\tilde{d}_R}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_{\text{Sd}}^2 \\ 0 & \epsilon_{\text{Sd}}^2 & 1 \end{pmatrix} m_0^2$$

$$M_{\tilde{L}}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_{\text{Su}}^2 \\ 0 & \epsilon_{\text{Su}}^2 & 1 \end{pmatrix} m_0^2, \qquad M_{\tilde{e}_R}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_{\text{Se}}^2 \\ 0 & \epsilon_{\text{Se}}^2 & 1 \end{pmatrix} m_0^2$$

$$a_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{22}^u \epsilon_{\text{Su}}^2 & 0 \\ 0 & 0 & c_{33}^u \end{pmatrix} A_0, \qquad a_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{22}^d \epsilon_{\text{Sd}}^2 & c_{23}^d \epsilon_{\text{Sd}}^2 \\ 0 & c_{23}^d \epsilon_{\text{Sd}}^2 & c_{33}^d \end{pmatrix} A_0,$$

$$a_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{22}^e \epsilon_{\text{Se}}^2 & c_{23}^e \epsilon_{\text{Se}}^2 \\ 0 & c_{23}^e \epsilon_{\text{Se}}^2 & c_{33}^e \end{pmatrix} A_0 \tag{5.8}$$

Note that for this case the trilinear couplings $\left(\sim z_{ij}^f \epsilon_f^p \right)$ will not be exactly aligned with the Yukawa couplings but will still be proportional to them and effectively we are taking $z_{ij}^f = c_{ij}^f \epsilon_{\text{Sd}}^2 / \epsilon_d^2$ for $(i, j) \neq (3, 3)$ and $z_{33}^f = c_{33}^f$.

6. Results

To get a feel for the range of parameter values of the coefficients c_{ij}^f and ϵ_u and ϵ_d we first consider a specific point in the susy parameter space with $m_{1/2} = 520\,\text{GeV}$, $m_0 = 330\,\text{GeV}$, $A_0 = 0$, and $\tan\beta = 50$. We also consider a case with $A_0 = -m_{1/2}$ and we restrict our attention to $\mu > 0$. In the CMSSM, this point corresponds to one lying on the WMAP co-annihilation strip [29] so the that the relic density lies within the WMAP preferred range. The Higgs mass is suitably large ($\gtrsim 116\,\text{GeV}$), the susy contribution to $b \to s\gamma$ is small, and the contribution to $(g - 2)_\mu$ accounts for the discrepancy between theory and experiment.

The goal of the analysis is to constrain the allowed parameter space both from flavour changing neutral currents and from the parameters determining the appropriate values of fermion masses at electroweak scale. As noted earlier, our free parameters are c_{22}^d and (ϵ_u, ϵ_d). We have set $c_{22}^d = 0.7$. The other c_{ij}^f are then fixed by the low energy fermion masses and mixings. Since we expect the parameters ϵ_u and ϵ_d to describe the structure of the Yukawa matrices, we do not want to rely on fine tuned coefficients c_{ij}^f. Indeed, V_{cb} would preclude a value of c_{23}^d as small as 0.1 if ϵ_d is also taken of order 0.1.

6.1 Running of off-diagonal soft parameters

Since we work in the context of the CMSSM, it is important to first establish the degree of running of our off-diagonal terms between the GUT and EW scales. The 1-loop RGE's for $(M_{\tilde{Q}}^2)_{ij}$, $(M_{\tilde{d}_R}^2)_{ij}$ and a_d are given by[1] [26]

$$
\begin{aligned}
\beta_{M_{\tilde{Q}}^2}^{(1)} &= \left(M_{\tilde{Q}}^2 + 2M_{H_u}^2 \right) Y_u^\dagger Y_u + \left(M_{\tilde{Q}}^2 + 2M_{H_d}^2 \right) Y_d^\dagger Y_d + \left[Y_u^\dagger Y_u + Y_d^\dagger Y_d \right] M_{\tilde{Q}}^2 \\
&\quad + 2Y_u^\dagger M_u^2 Y_u + 2Y_d^\dagger M_d^2 Y_d + 2a_d^\dagger a_d \\
&\quad - \frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{9} g_1^2 M_1^2 + \frac{1}{5} g_1^2 \mathcal{S} \\
\beta_{M_{\tilde{d}}^2}^{(1)} &= \left(2M_{\tilde{d}}^2 + 4M_{H_d}^2 \right) Y_d Y_d^\dagger + 4Y_d M_{\tilde{Q}}^2 Y_d^\dagger + 2Y_d Y_d^\dagger M_{\tilde{d}}^2 \\
&\quad + 4a_d a_d^\dagger - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{9} g_1^2 |M_1|^2 + \frac{2}{3} g_1^2 \mathcal{S} \\
\beta_{a_d}^{(1)} &= a_d \left\{ \mathrm{Tr} \left(3Y_d Y_d^\dagger + Y_e Y_e^\dagger \right) + 5Y_d^\dagger Y_d Y_u^\dagger Y_u - \sum_{a=1,2,3} u_a g_a^2 \right\} \\
&\quad + Y_d \left\{ \mathrm{Tr} \left(6a_d Y_d^\dagger + 2a_e Y_e^\dagger \right) + 4Y_d^\dagger a_d + 2Y_u^\dagger a_u - 2 \sum_{a=1,2,3} u_a g_a^2 M_a \right\}
\end{aligned}
\tag{6.1}
$$

where $u_a = 7/9, 3, 16/3$ are numerical coefficients, g_a are the gauge couplings and M_a are the masses of the gauginos. We have not shown (though we do include them in our calculations) the beta functions for $M_{\tilde{u}}^2$, $M_{\tilde{L}}^2$, and $M_{\tilde{e}}^2$. $\beta_{M_{\tilde{u}}^2}^{(1)}$ is similar to $\beta_{M_{\tilde{d}}^2}^{(1)}$ with the interchange $u \leftrightarrow d$ and different contributions from the gauginos. The running in the $\tilde{u}$ sector is in general small due to our assumption that the expansion parameter in the up sector is small in comparison with that of the down sector. The running of $M_{\tilde{L}}^2$ ($M_{\tilde{e}}^2$) is similar to that of $M_{\tilde{Q}}^2$ ($M_{\tilde{d}}^2$) due to their common dependence on ϵ_u (ϵ_d). Here, we will describe the running of $(M_{\tilde{Q}}^2)_{ij}$, $(M_{\tilde{d}_R}^2)_{ij}$ and a_d, but the running for $(M_{\tilde{L}}^2)_{ij}$, $(M_{\tilde{e}_R}^2)_{ij}$ and a_e, has a similar behaviour.

Y_d is not diagonal at the GUT scale, however, Y_u is, and although at two loops the evolution of Y_u produces off-diagonal elements we neglect them here, due to the smallness of ϵ_u. Therefore, the most important contributions to the beta functions of the off-diagonal terms of the the soft squared masses in eq. (6.1) are found in the terms

$$
\left(M_{\tilde{Q}}^2 Y_f^\dagger Y_f \right)_{ij}, \quad \left(Y_f^\dagger Y_f M_{\tilde{Q}}^2 \right)_{ij}, \quad \left(Y_f^\dagger M_{\tilde{f}}^2 Y_f \right)_{ij}, \quad \left(a_d^\dagger a_d \right)_{ij}
\tag{6.2}
$$

For the off-diagonal trilinear terms, important contributions arise from the terms:

$$
\left(a_d Y_d^\dagger Y_d \right)_{ij}, \quad \left(Y_d Y_d^\dagger a_d \right)_{ij}, \quad \left(Y_d \mathrm{Tr} \left(a_d Y_d^\dagger \right) \right)_{ij}, \quad \left(Y_d \mathrm{Tr} \left(a_e Y_e^\dagger \right) \right)_{ij}.
\tag{6.3}
$$

---

[1]Note that we employ a sign convention for a which differs by a relative sign from that of [26].

Of course we need only consider the case for $f = d$ and $(i,j) = (2,3)$. Expressing Y_d in terms of ϵ_d, the contribution to the beta functions of the soft squared masses is

$$\left(M_{\tilde{Q}}^2 Y_d^\dagger Y_d\right)_{23} = \left[c_{23}^d c_{33}^d \epsilon_d^2 M_{\tilde{Q}\ 22}^2 + c_{33}^{d\ 2} M_{\tilde{Q}\ 23}^2\right] \to c_{23}^d c_{33}^d M_{\tilde{Q}\ 22}^2 \epsilon_d^2 = c_{23}^d c_{33}^d \epsilon_d^2 m_0^2$$

$$\left(Y_d^\dagger Y_d M_{\tilde{Q}}^2\right)_{23} = \left[\left(c_{22}^{d\ 2} + c_{23}^{d\ 2}\right)\epsilon_d^2 M_{\tilde{Q}\ 23}^2 + c_{23}^d c_{33}^d M_{\tilde{Q}\ 33}^2\right]\epsilon_d^2 \to c_{23}^d c_{33}^d M_{\tilde{Q}\ 33}^2 \epsilon_d^2 = c_{23}^d c_{33}^d \epsilon_d^2 m_0^2$$

$$\left(Y_d^\dagger M_{\tilde{f}}^2 Y_d\right)_{23} = \left[c_{22}^d c_{23}^d M_{\tilde{d}\ 22}^2 + c_{23}^{d\ 2} M_{\tilde{d}\ 32}^2\right]\epsilon_d^4 + \left[c_{22}^d c_{33}^d M_{\tilde{d}\ 23}^2 + c_{23}^d c_{33}^d M_{\tilde{d}\ 33}^2\right]\epsilon_d^2$$
$$\to c_{22}^d c_{23}^d M_{\tilde{d}\ 22}^2 \epsilon_d^4 + c_{23}^d c_{33}^d M_{\tilde{d}\ 33}^2 \epsilon_d^2 \simeq c_{23}^d c_{33}^d \epsilon_d^2 m_0^2$$

$$\left(a_d^\dagger a_d\right)_{23} = z_{23}^d z_{33}^d \epsilon_d^2 A_0^2 \to 0 \tag{6.4}$$

where we have kept only like terms to lowest order in ϵ_d. The expressions on the right-hand side of the arrows are the remaining non-zero contributions when the off-diagonal terms in eqs. (2.2) and (2.3) are set to zero. From eq. (5.7) we can see that since $M_{\tilde{Q}\ 22}^2 \gg M_{\tilde{Q}\ 23}^2$, even when the off-diagonal terms of eq. (6.2) are set to zero, the leading terms $\sim M_{\tilde{Q}\ ii}^2 \epsilon_d^2$, of eq. (6.4) remain, except for the term $a_d^\dagger a_d$, which vanishes. For the contribution to the beta functions of the trilinear couplings we have

$$\left(a_d Y_d^\dagger Y_d\right)_{23} = \sum_l \left[z_{2l}^d \epsilon_d^2\right] A_0 \left(Y_d^\dagger Y_d\right)_{l3} \to z_{22}^d c_{23}^d c_{33}^d \epsilon_d^4 A_0$$

$$\left(Y_d Y_d^\dagger a_d\right)_{23} = z_{23}^d \epsilon_d^2 A_0 \left(Y_d^\dagger Y_d\right)_{22} + z_{33}^d A_0 \left(Y_d^\dagger Y_d\right)_{23} \to z_{33}^d c_{23}^d c_{33}^d \epsilon_d^2 A_0$$

$$\left[Y_d \mathrm{Tr}\left(a_d Y_d^\dagger\right)\right]_{23} = c_{22}^d c_{23}^d \epsilon_d^6 z_{22}^d A_0 + 2c_{23}^{d\ 2} \epsilon_d^6 z_{23}^d A_0 + c_{23}^d c_{33}^d \epsilon_d^2 z_{33}^d A_0$$
$$\to \left[c_{22}^d c_{23}^d \epsilon_d^6 z_{22}^d + c_{23}^d c_{33}^d \epsilon_d^2 z_{33}^d\right] A_0$$

$$\left[Y_d \mathrm{Tr}\left(a_e Y_e^\dagger\right)\right]_{23} = c_{22}^e c_{23}^d \epsilon_d^6 z_{22}^e A_0 + 2c_{23}^d c_{23}^e \epsilon_d^6 z_{23}^e A_0 + c_{23}^d c_{33}^e \epsilon_d^2 z_{33}^e A_0$$
$$\to \left[c_{22}^e c_{23}^d \epsilon_d^6 z_{22}^e + c_{23}^d c_{33}^e \epsilon_d^2 z_{33}^e\right] A_0 \tag{6.5}$$

In eq. (6.5) we see again that when the off-diagonal terms of the trilinear couplings $a_{d,e}$ are set to zero the leading terms of the original expression do not vanish.

In figure 1, we show the running of the off-diagonal soft masses. We begin by discussing the case for $A_0 = 0$. Here, we have chosen $\epsilon_d = 0.1$ and $\epsilon_u = 0.05$ as well as $c_{22}^d = 0.7$ which will be used throughout this section. The running of the soft masses in this case is shown by the dashed (when the off-diagonal terms of the soft squared masses are set to zero at the GUT scale) and solid (when they are non-zero) blue curves. In the first panel of figure 1, we show the running of $M_{\tilde{Q}\ 23}^2$ as a function of the renormalization scale Q. When the off-diagonal terms are turned off, the running starts at $M_{\tilde{Q}\ 23}^2 = 0$ as seen by the dashed (blue) curve. The dominant contributions to the running of are suppressed by ϵ_d^2 and proportional to $c_{23}^d c_{33}^d$. As a consequence, the running of $M_{\tilde{Q}\ 23}^2$ is relatively significant. When the off-diagonal elements are turned on at the GUT scale, the initial value for $M_{\tilde{Q}\ 23}^2$ is about $(16.5\,\mathrm{GeV})^2$ (as seen more clearly in panel b for $M_{\tilde{u}\ 23}^2$). On this scale, the difference in the running is relatively minor as all of the leading contributions to these expressions are the same. As a result the evolution of these terms does not change drastically according to the different initial conditions at the GUT scale.
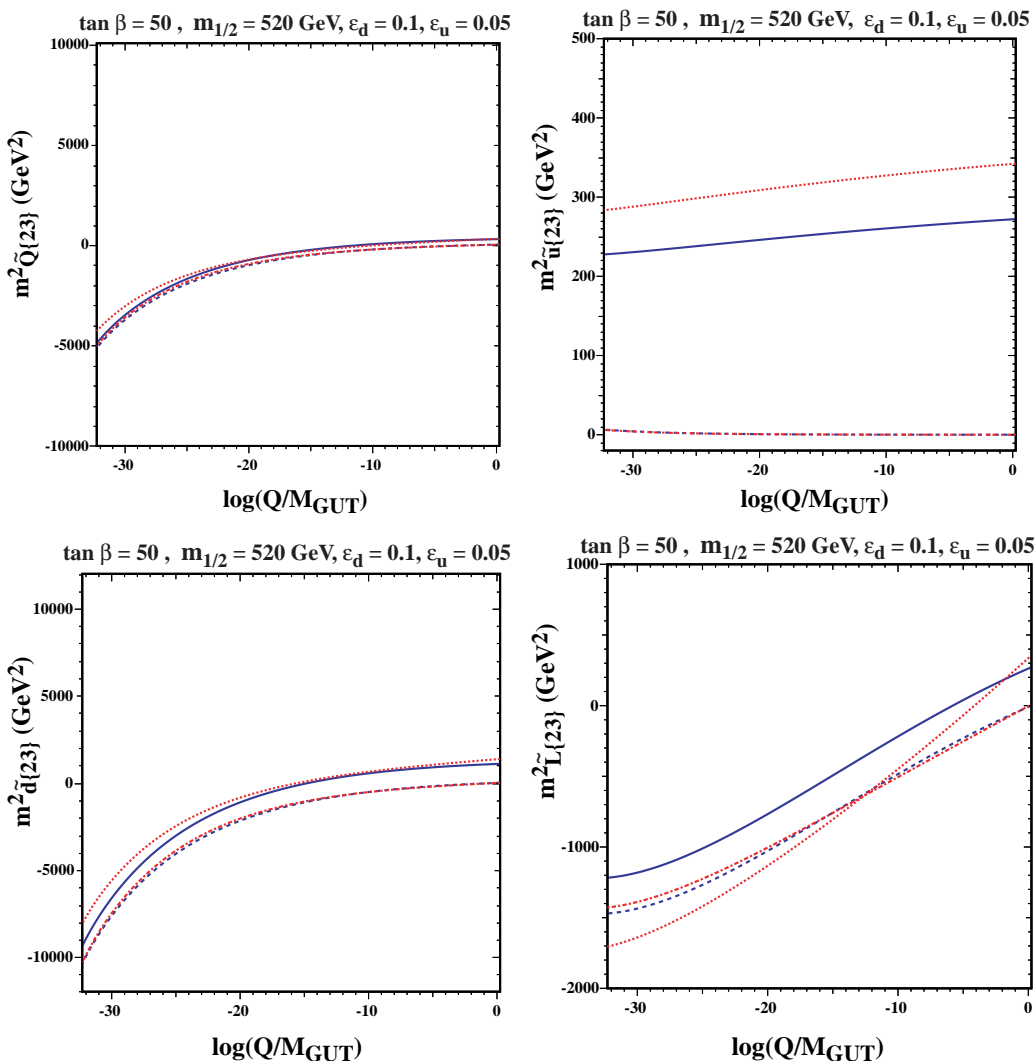
Figure 1: The running of the parameters of the soft squared masses a) $M^2_{\tilde{Q}\ 23}$, b) $M^2_{\tilde{u}\ 23}$, c) $M^2_{\tilde{d}\ 23}$, and d) $M^2_{\tilde{L}\ 23}$ when off-diagonal terms are turned off (dashed blue) and on (solid blue) at the GUT scale with $A_0 = 0$. Here $m_{1/2} = 520\,\mathrm{GeV}$, $m_0 = 330\,\mathrm{GeV}$, $\epsilon_d = 0.1$ and $\epsilon_u = 0.05$. Also shown is the running when $A_0 = -m_{1/2}$ and $m_0 = 370\,\mathrm{GeV}$. The dot-dashed (red) curve corresponds to the case where off-diagonal terms are turned off at the GUT scale and the dotted (red) curve corresponds to the case where they are turned on.

Also shown in figure 1a is the analogous running when $A_0 \neq 0$. Here we show the running for the specific choice $A_0 = -m_{1/2} = -520\,\mathrm{GeV}$. Note that for this case, we have adjusted m_0 to $370\,\mathrm{GeV}$ in order to maintain the correct relic density. When $A_0 \neq 0$, the evolution of soft squared masses will have contributions from the terms $a^\dagger_f a_f$ when the off-diagonal terms start with a non-zero value at the GUT scale. As the dominant contribution to the running of $M^2_{\tilde{Q}\ 23}$ comes from the gaugino (mostly gluino) terms in eq. (6.1), $a^\dagger_f a_f$ tends to lower (slightly) the running as seen by the dash-dotted (when off-diagonal terms
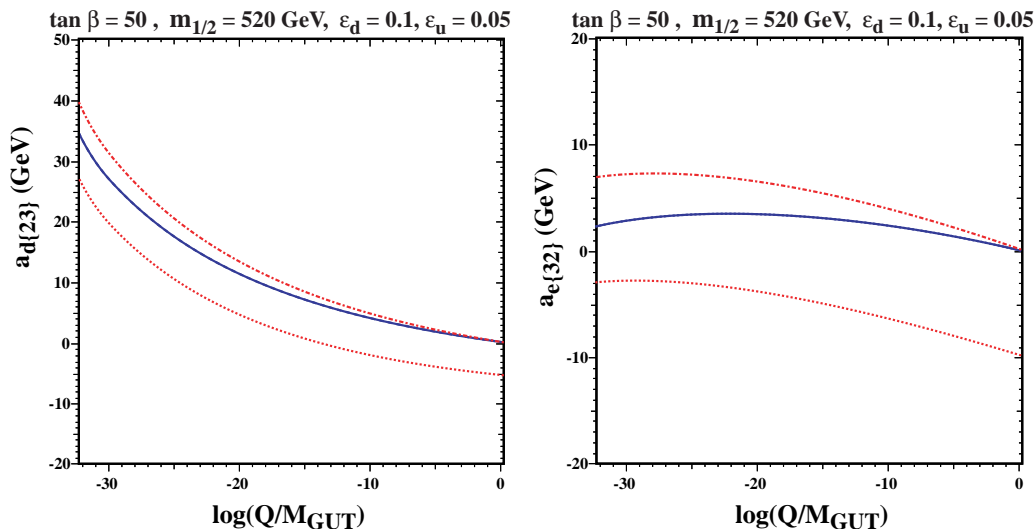
Figure 2: As in figure 1 for the running of a) a_{d23} and b) a_{e32}.

are turned off at the GUT scale) and dotted (when they are turned on) (red) curves.

In contrast, in panel b) of figure 1, we show the running of $M^2_{\tilde{u}\ 23}$. The beta function for $M^2_{\tilde{u}\ 23}$ is determined by Y_u or a_u rather than Y_d and as a consequence the running is minimal. As expected from the beta functions, the running of $M^2_{\tilde{d}\ 23}$ is also significant as seen in panel c). We also show the running of $M^2_{\tilde{L}\ 23}$ in panel d). As noted earlier, the beta function for $M^2_{\tilde{L}\ 23}$ is similar to that of $M^2_{\tilde{Q}\ 23}$ though the running is smaller due to the lack of gluino contributions.

Although the running of $(M^2_d)_{23}$ is the most significant of those shown in figure 1, in the computation of BR(b $\rightarrow$ s γ), discussed below, this effect is sub-dominant. The reason is that the contributions from the charged Higgs boson and the charginos is much more important than the contribution from gluinos. We can see from eq. (B.16) that only the diagonalization matrix of $M^2_{\tilde{u}}$, K^u, is relevant for the computation of the magnetic Wilson coefficients where the charginos and charged Higgs boson are the virtual particles. The diagonalization matrix of $M^2_{\tilde{d}}$, K^d, is relevant only for the gluino contributions. This is because they enter into the computation of chromo-magnetic Wilson coefficients arising from the virtual exchange of gluinos [14].

For $A_0 = 0$ the renormalization group equations generate a contribution for a_d of order $-Y_d \sum_a u_a M_a$ at one loop. In addition, when $A_0 = 0$, the running of a_d is independent of whether off-diagonal terms of soft parameters are set to zero at the GUT scale. In figure 2, we show the running of a_{d23} in panel a) and a_{e23} in panel b). The solid (blue) line corresponds to the case where $A_0 = 0$. In both cases the running is very slight.

When $A_0 \neq 0$, the first term of the one loop beta function of a_d will now be relevant. However since the off-diagonal terms $(a_f)_{ij}$ are suppressed with respect to the diagonal terms $(a_f)_{ii}$ as $(a_f)_{ij}/(a_f)_{ii} = \epsilon_d^2$, for $i,j = 2,3$ and $f = d,e$, the difference will be small. The running of a_{d23} and a_{e23} with $A_0 = -m_{1/2}$ is also shown in figure 2.

In order to determine the possible magnification of the flavour violating parameters at the EW scale given the GUT scale initial conditions eq. (5.7), it will be useful to define the parameters,

$$(\delta_{f,XY})_{ij} = \frac{(M^2_{f,XY})_{ij}}{\sqrt{(M^2_{f,XX})_{ii}(M^2_{f,YY})_{jj}}} \tag{6.6}$$

where $(M^2_{f,XY})_{ij}$, $X,Y = L,R$ are the running soft sfermion parameters. In order to give meaning to the flavour violating parameters, we must work in the basis where fermion masses are diagonal. The details of this rotation are given in appendix B (see: eq. (B.6)). In the definition of the δ's, all soft masses will be assumed to be in the rotated basis.

In figure 3a, we show the running of $M^2_{\tilde{d}\,23}$ in the rotated basis. Since $M^2_{\tilde{d}}$ depends primarily on $(Y^\dagger_d Y_d)_{23} \propto \epsilon^2_d$ (if $\epsilon^2_u \ll \epsilon^2_d$) when the transformation to the rotated basis is performed, we expect that the value of $M^2_{\tilde{d}\,23}$ will be reduced due to its alignment with the Yukawa matrix Y_d. Additionally, for $A_0 = 0$, when off-diagonal terms are turned off, there is no contribution from the trilinear terms to the running of soft squared masses, as shown in eq. (6.4). Thus, when going to the basis where Yukawa couplings are diagonal ($|\hat{Y}_f|^2 = V^{f\mathrm{T}}_R Y^\dagger_f Y_f V^{f*}_R$), the contributions of the form $Y^\dagger_f Y_f$ and $Y_f Y^\dagger_f$ will also be rotated away to a large extent. We can see then in figure 3a, that this is indeed the case for $A_0 = 0$, $\epsilon_d = 0.1$ (blue-dashed line). When $A_0 \neq 0$, we see from eq. (6.4) that we have an extra contribution to the running of $M^2_{\tilde{d}}$ from the trilinear terms, which cannot be greatly reduced when going to the rotated basis. When off-diagonal terms are turned on, for the case of $A_0 = 0$ we expect a greater effect in the running of $M^2_{\tilde{d}}$ than in any of the other soft squared masses, because (along with $M^2_{\tilde{e}}$) this is the only matrix that is sensitive to the parameter ϵ_d at the GUT scale. In figure 3a, the (green) long dashed curve corresponds to $\epsilon_d = 0.16$ while the (blue) solid curve for $\epsilon_d = 0.1$. Comparing the ratio $0.1^2/0.16^2 = 0.39$ we see that in the un-rotated basis (figure 1c) the difference between the two initial conditions for $M^2_{\tilde{d}}$ will be 39% and is increased to about 50% in the rotated basis because the diagonalization matrix also contains the parameter ϵ^2_d.

The sensitivity to ϵ_u in the running of $M^2_{\tilde{d}}$ is very small as a result of the difference between ϵ^2_d and ϵ^2_u. The sensitivity to c^d_{22} in the running of the soft squared masses comes from the terms of the form $Y_f Y^\dagger_f$ and $a_f a^\dagger_f$, however as we will describe in the next section, the elements of $Y_{f_{ij}}$ are adjusted to reproduce the correct fermion masses at the EW scale for a given choice of c^d_{22} and ϵ_d. For $c^d_{22} = 0.7$ and $\epsilon_d = 0.1$, one is able to reproduce the correct value of m_s(EW). Similarly, for $c^d_{22} = 0.5$ and $\epsilon_d = 0.12$ we are also able to obtain the correct value for m_s(EW). Indeed, the product $c^d_{22}\epsilon^2_d$ at the GUT scale is very similar. Therefore, the quantities $Y_f Y^\dagger_f$ are more or less fixed by this requirement for different choices of c^d_{22} and ϵ^2_d and as a result the running of $M^2_{\tilde{d}}$ will not be very sensitive to these changes.

In the remaining panels of figure 3, we show the flavour violating parameters $(\delta_{q,XX})_{23}$, $(\delta_{d,XY})_{23}$, and $(\delta_{e,XX})_{23}$ for our test point $(m_{1/2}, m_0, A_0) = (520, 330, 0)$ GeV and $(\epsilon_d, \epsilon_u) = (0.1, 0.05)$. The curves for these cases are all shown as solid (blue). Off-diagonal terms at the GUT scale are turned on for all curves in panels b) - f). For comparison, we show the case with non-zero A_0, ie., $(m_{1/2}, m_0, A_0) = (520, 370, -520)$ GeV. This case is
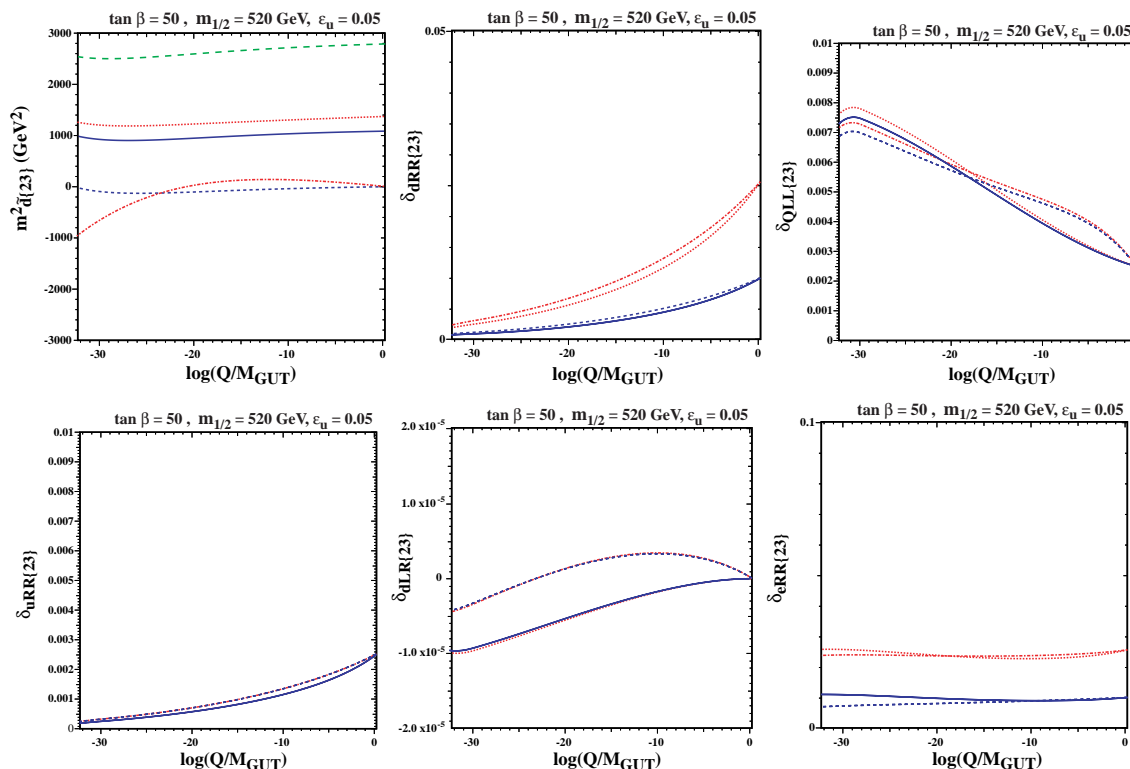
Figure 3: In panel a), the running of $M^2_{\tilde{d}\ 23}$ in the rotated basis as in figure 1. Also shown by the (green) long-dashed curve is the case $\epsilon_d = 0.16$ with $A_0 = 0$ and off-diagonal terms turned on at the GUT scale. In the remaining panels, the running of squark flavour violating parameters $(\delta_{q,XX})_{23}$ (b-d), $(\delta_{d,XY})_{23}$ (e), and $(\delta_{e,XX})_{23}$ (f) for $(m_{1/2}, m_0, A_0) = (520, 330, 0)\,\mathrm{GeV}$, with $\tan\beta = 50$, $\epsilon_d = 0.1$ and $\epsilon_u = 0.05$ shown by the solid (blue) curves. In panels b) -f), off-diagonal terms are on at the GUT scale. Also shown by the dashed (blue) curves are the results for $A_0 = -m_{1/2}$ with $m_0 = 370\,\mathrm{GeV}$. The (red) dotted and dash-dotted curves show results for $\epsilon_d = 0.16$ with $A_0 = 0$ and $A_0 = -520\,\mathrm{GeV}$ respectively.

shown by the dashed (blue) curves. In addition for both $A_0 = 0$ and $A_0 \neq 0$, we show the sensitivity of the running to ϵ_d. The (red) dotted and dash-dotted curves correspond to these cases respectively, where we have chosen $\epsilon_d = 0.16$.

6.2 Determination of coefficients of Yukawa matrices

In this subsection, we will determine the values and sensitivities of the quantities

$$c^d_{23}, \quad c^d_{33}, \quad c^e_{22}, \quad c^e_{23}, \quad c^e_{33}, \quad c^u_{22} \quad \text{and} \quad c^u_{33} \tag{6.7}$$

to the variation of ϵ_d and ϵ_u for a given value c^d_{22}. For now, we continue to use our example based on the point $(m_{1/2}, m_0, A_0) = (520, 330, 0)$ GeV, $\tan\beta = 50$. As a result of the small differences in the running of the off-diagonal terms of soft parameters when we choose different initial conditions at the GUT scale, the Yukawa parameters are not really sensitive to this difference. In this section, we will only consider cases where the
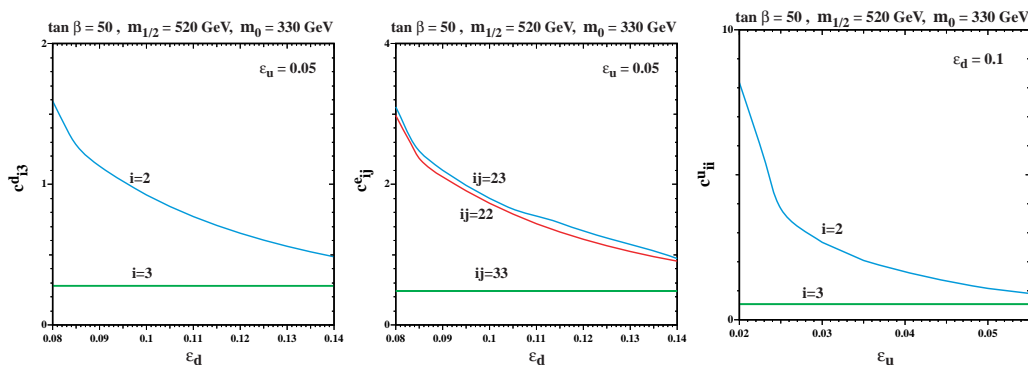
Figure 4: Yukawa parameters, c_{ij}^f of the d, e and u sectors for the supersymmetric point $(m_{1/2}, m_0, A_0) = (520, 330, 0)\,\mathrm{GeV}$, $\tan\beta = 50$ and $c_{22}^d = 0.7$. In panel (a) we show c_{i3}^d for $i = 2, 3$ as labeled as a function of ϵ_d for fixed $\epsilon_u = 0.05$. In panel (b) we show c_{ij}^e for $ij = 22, 23, 33$ as labeled as a function of ϵ_d for fixed $\epsilon_u = 0.05$. Finally in panel (c) we show c_{ii}^u for $i = 2, 3$ as labeled as a function of ϵ_u for fixed $\epsilon_d = 0.1$.

off-diagonal elements are turned on at the GUT scale. These parameters are, however, sensitive to variations in ϵ_d, ϵ_u and c_{22}^d.

In figure 4, we plot the relevant c_{ij}^f versus the appropriate ϵ_f for $c_{22}^d = 0.7$. There is little sensitivity to ϵ_d or ϵ_u for the 33 components of the Yukawa matrices which are well fixed by the low energy fermion masses. In contrast, the 22 and 23 components can be sensitive to ϵ_d and ϵ_u even within their limited range of variation.

In figure 5, we show the dependence of two of the Yukawa parameters, c_{23}^d and c_{33}^d, to our choice of the supersymmetric point as a function of ϵ_d. In addition to our original test point (shown here as the dotted curves), we consider $(m_{1/2}, m_0, A_0) = (160, 630, 0)$ GeV shown by the dashed curves, and $(m_{1/2}, m_0, A_0) = (1240, 910, -1240)$ GeV shown by the solid curves. In all three cases, we have chosen $\tan\beta = 50$. In each case, the value of m_0 was chosen so as to obtain a relic density compatible with WMAP. In the latter case, we chose $A_0 = -m_{1/2}$. The dependence of the parameters of Y_e on the change of $(m_{1/2}, m_0, A_0)$ is similar though less pronounced and the dependence of the parameters of Y_u is very small.

Given the parameterization of the Yukawa matrix Y_d in eq. (5.2), the choice of c_{22}^d is directly correlated to the allowed value of ϵ_d given the value of $m_s/m_b(M_{\mathrm{GUT}})$:

$$\frac{m_s}{m_b}\left(M_{\mathrm{GUT}}\right) \sim \frac{(Y_d)_{22}}{(Y_d)_{33}} - \frac{(Y_d^2)_{22} + (Y_d^2)_{33}}{2(Y_d^2)_{33}} = \frac{c_{22}^d}{c_{33}^d}\epsilon_d^2 - O(\epsilon_d^4), \tag{6.8}$$

which must be in the range $\sim (0.01, 0.03)$ at M_{GUT}. Hence for different choices of c_{22}^d, assumed to be $\mathcal{O}(1)$, we have the following allowed ranges of ϵ_d:

$$
\begin{aligned}
c_{22}^d = 0.5 &\;\rightarrow\; \epsilon_d \simeq [0.10, 0.12] \\
c_{22}^d = 0.7 &\;\rightarrow\; \epsilon_d \simeq [0.085, 0.10] \\
c_{22}^d = 1.0 &\;\rightarrow\; \epsilon_d \simeq 0.08.
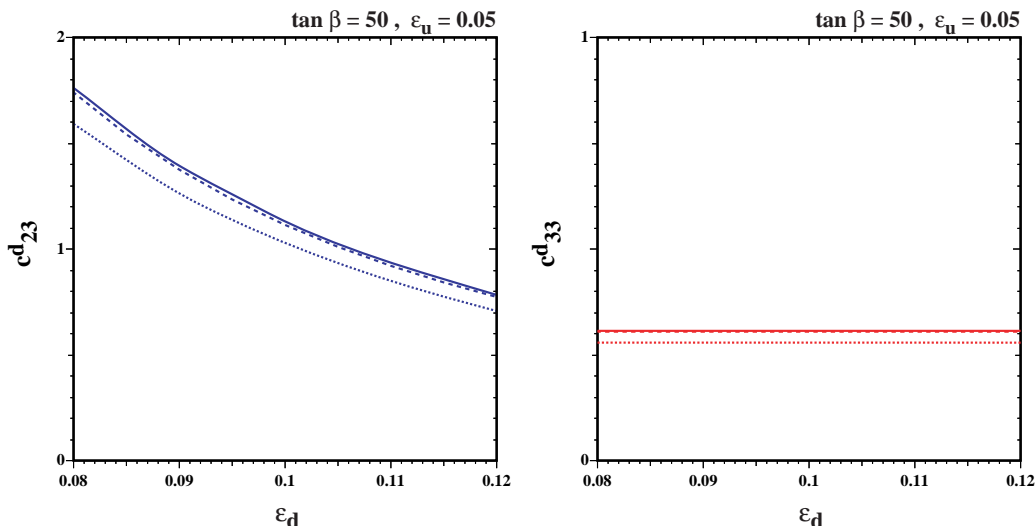\end{aligned}
\tag{6.9}
$$

Figure 5: As in figure 4, we plot (a) c_{23}^d and (b) c_{33}^d for three different choices of $(m_{1/2}, m_0, A_0)$. The bottom curves (dotted) correspond to $(m_{1/2}, m_0, A_0) = (520, 330, 0)$ GeV, the lines in the middle (dashed) to $(m_{1/2}, m_0, A_0) = (160, 630, 0)$ GeV and the lines at the top (solid) to $(m_{1/2}, m_0, A_0) = (1240, 910, -1240)$ GeV.

Yukawa parameters c_{ij}^f					
c_{22}^d	c_{23}^d	c_{33}^d	c_{22}^e	c_{23}^e	c_{33}^e
0.5	[0.6, 0.9]	0.3	[1.2, 1.7]	[1.3, 1.7]	0.5
0.7	[0.9, 1.3]	0.3	[1.7, 2.4]	[1.9, 2.4]	0.5
1.0	1.6	0.3	3.0	3.1	0.5

Table 1: Values of the Yukawa parameters for different choices of c_{22}^d for the supersymmetric point $(m_{1/2}, m_0, A_0) = (520, 330, 0)$ GeV.

As a consequence the values of $(Y_d)_{ij}$ and $(Y_e)_{ij}$ are different for each case. In table 1, we show the allowed ranges of the Yukawa parameters according to the allowed range for ϵ_d in each case.

To further compare how the determination of the appropriate Yukawa parameters at the GUT scale depends on the chosen supersymmetric parameters we show in figure 6, the values of c_{ij}^f as a function of the the gaugino mass, $m_{1/2}$. Here $A_0 = 0$ and $\tan \beta = 50$. For each value of $m_{1/2}$, m_0 is adjusted to insure the correct relic density of neutralinos. As one can see the dependence on the supersymmetric model is rather minimal.

Finally, we would like to emphasize the importance of taking into account the supersymmetric corrections to the fermion masses for the proper determination of Yukawa parameters at the GUT scale. From [39], we find that for large $\tan \beta$, the supersymmetric corrections to the mass of the bottom can be as large as 30%, as it is for the case of $\tan \beta = 50$. We can express the relation between the masses of the fermions at a certain scale Q, containing the supersymmetric corrections, and the masses of fermions at the same
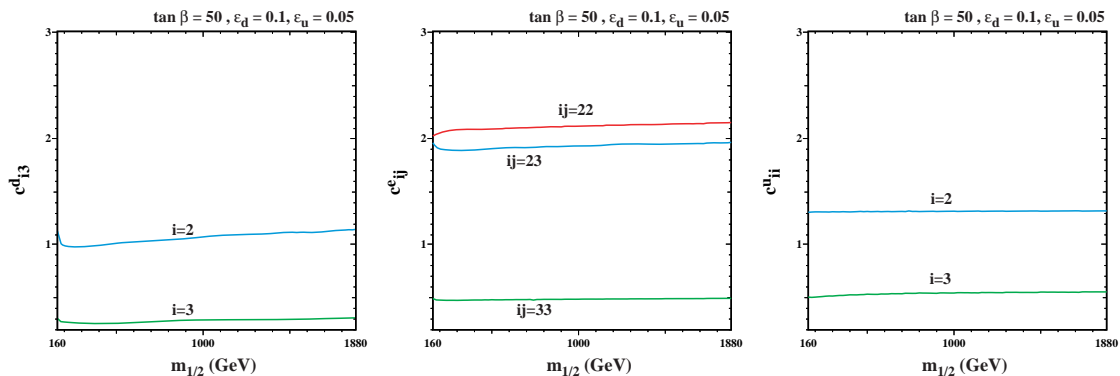
Figure 6: The coefficients of the Yukawa matrices plotted versus $m_{1/2}$ for $\epsilon_d = 0.1$, $\epsilon_u = 0.05$ and $c_{22}^d = 0.7$. In panel (a) we show c_{i3}^d for $i = 2, 3$ as labeled. In panel (b) we show c_{ij}^e for $ij = 22, 23, 33$ as labeled. Finally in panel (c) we show c_{ii}^u for $i = 2, 3$ as labeled.

scale as given by the SM through:

$$m_k^S(Q) = m_k^{\text{SM}}(Q) g_k^S(Q), \quad k = t, b, c, s. \tag{6.10}$$

Here $m_k^{\text{SM}}(Q)$ are the SM values at Q, including the corrections from gluons. A good analytical approximation to the factors $g_k^S(Q)$ is given in [39]. For example, for the top quark, this can be expressed as $g_t^S(Q) = 1 + \Delta m_t/m_t$, where the most important contributions to $\Delta m_t/m_t$ comes from top squark/gluino loops. In figure 7, we plot the numerical values of $g_k^S(Q)$ for $k = t, b, s$, as a function of $m_{1/2}$ for $\tan\beta = 50$ and for the choice of Yukawa parameters $(c_{22}^d, \epsilon_d) = (0.7, 0.1)$. These corrections include the corrections at one loop from squark/gluino loops, charginos and neutralinos. We can see that, in fact, for m_b these corrections are about 30%, while for m_s, they can be up to 14%. For m_t, the corrections are within 11%. The corrections for m_c, though included are not shown as they are very small.

The supersymmetric corrections change the profile of Yukawa matrices at the GUT scale. A SM analysis [2] determined the values for Y_d (for symmetric matrices) as follows:

$$\frac{(Y_d)_{22}}{(Y_d)_{33}} = 0.15^2 = 0.0225, \quad \frac{(Y_d)_{23}}{(Y_d)_{33}} = 1.3 \left(0.15^2\right) = 0.0293. \tag{6.11}$$

If we take the values of the Yukawa parameters for $\left(c_{22}^d, \epsilon_d\right) = (0.7, 0.1)$ from table 1, we have

$$\frac{(Y_d)_{22}}{(Y_d)_{33}} = 0.025, \quad \frac{(Y_d)_{23}}{(Y_d)_{33}} = 0.045. \tag{6.12}$$

Since c_{22}^d is an input to our analysis, we are basically adjusting the value of $(Y_d)_{22}/(Y_d)_{33}$, at the GUT scale, thus there is no surprise in the similar values for this ratio in both cases. What is interesting is the change in the off-diagonal parameter $(Y_d)_{23}/(Y_d)_{33}$, since this off-diagonal term is crucial when constructing a specific flavour symmetry.

It is important to emphasize the difference between the structure of the Yukawa matrices at the GUT and EW scales. Whilst at the GUT scale they can be written in terms
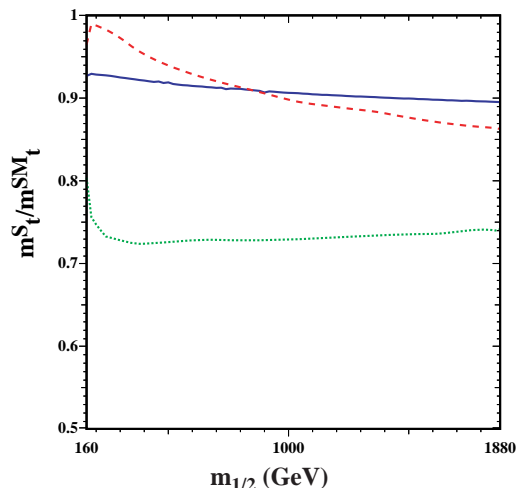
Figure 7: MSSM corrections at one loop to the masses of m_t (solid blue line), m_b (dotted green line) and m_s (dashed red line).

of the expansion parameter ϵ_d^2 and the prefactors c_{ij}^f, at the EW scale this is not longer the case. However if the prefactors c_{ij}^f are indeed to be explained by a specific FS then the result at the EW can be expressed as a series expansion in terms of ϵ_f and the same coefficients. For example, we can express the Yukawa matrices normalized to the elements $(Y_f)_{33}$ at the GUT scale. The numerical value using $c_{22}^d = 0.7$, $\epsilon_d = 0.1$ and $\epsilon_u = 0.05$ is

$$
\frac{Y_d}{(Y_d)_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{c_{22}^d}{c_{33}^d}\epsilon_d^2 & \frac{c_{23}^d}{c_{33}^d}\epsilon_d^2 \\ 0 & \frac{c_{23}^d}{c_{33}^d}\epsilon_d^2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.025 & 0.033 \\ 0 & 0.033 & 1 \end{pmatrix}
$$

$$
\rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.07 & 0.04 \\ 0 & 0.04 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \left(\frac{c_{22}^d}{c_{33}^d} + 4.49\right)\epsilon_d^2 & \frac{c_{23}^d}{c_{33}^d}\epsilon_d^2 + 7\epsilon_d^3 \\ 0 & \frac{c_{23}^d}{c_{33}^d}\epsilon_d^2 + 7\epsilon_d^3 & 1 \end{pmatrix}
$$

$$
\frac{Y_e}{(Y_e)_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{c_{22}^e}{c_{33}^e}\epsilon_d^2 & \frac{c_{23}^e}{c_{33}^e}\epsilon_d^2 \\ 0 & \frac{c_{23}^e}{c_{33}^e}\epsilon_d^2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.038 & 0.036 \\ 0 & 0.036 & 1 \end{pmatrix}
$$

$$
\rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.05 & 0.04 \\ 0 & 0.04 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \left(\frac{c_{22}^e}{c_{33}^e} + 1.25\right)\epsilon_d^2 & \frac{c_{23}^d}{c_{33}^d}\epsilon_d^2 + 3.9\epsilon_d^3 \\ 0 & \frac{c_{23}^d}{c_{33}^d}\epsilon_d^2 + 3.9\epsilon_d^3 & 1 \end{pmatrix}
$$

$$
\frac{Y_u}{(Y_u)_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{c_{22}^u}{c_{33}^u}\epsilon_u^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.005 & 0 \\ 0 & 0 & 1 \end{pmatrix}
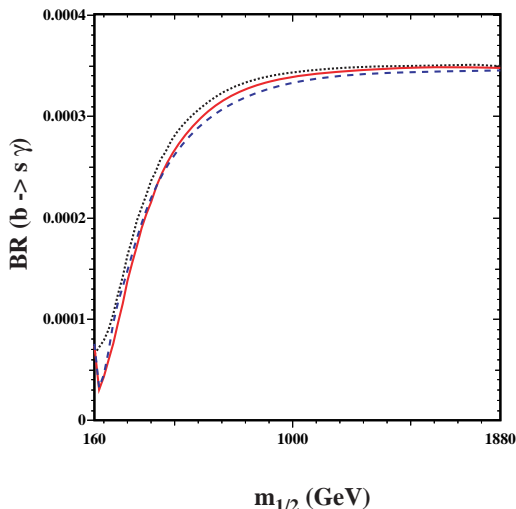$$

– 22 –

Figure 8: The branching ratio for $b \rightarrow s\,\gamma$ for $A_0 = 0$ (solid red) and $A_0 = -m_{1/2}$ (dotted black) when $\epsilon_d = 0.1$, $\epsilon_d = 0.05$ and $c_{22}^d = 0.7$. When only the running of the 3rd generation is included, the analogous result is shown by the dashed (blue) curve.

$$\rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \left(\frac{c_{22}^u}{c_{33}^u} + 2\right)\epsilon_u^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6.13}$$

where the effect of running to the EW scale is given to the right of the arrows.

6.3 Effects on $b \rightarrow s\,\gamma$

We have seen that a good choice of Yukawa parameters was $c_{22}^d = 0.7$ and $\epsilon_d = 0.1$, $\epsilon_u = 0.05$, thus we present the analysis on the sensitivity of $b \rightarrow s\,\gamma$ on ϵ_d centered on those parameter values. As we explain in appendix A due to the hierarchy of $\epsilon_u^2 \ll \epsilon_d^2$, we expect that the main departures from MFV come from the gluino contributions, entering through the magnetic Wilson coefficients due to the diagonalisation of the soft squared matrices in the $\tilde{d}$ sector, eq. (5.7). In figure 8 we show our calculation of $b \rightarrow s\,\gamma$ as a function of $m_{1/2}$ when the running of the parameters of the second family have not been considered (dashed blue curve). For comparison, we also show the same result when the running of both the 2nd and 3rd generations are included for $A_0 = 0$ (solid red curve) and when $A_0 = -m_{1/2}$ (dotted black curve). As before, for each value of $m_{1/2}$, m_0 is chosen so as to obtain the WMAP value for the relic density of neutralinos. Note that the inclusion of the 2nd generation running relaxes slightly the bounds obtainable from a comparison of the computed value of $b \rightarrow s\,\gamma$ with the experimental measurement at relatively low $m_{1/2}(\sim 400\,\text{GeV})$. As long as we fix the relation between c_{22}^d and ϵ_d through the fermion masses, there will be little sensitivity of $b \rightarrow s\,\gamma$ to ϵ_d.

The contribution of the charged Higgs to the Wilson coefficients, depends crucially on $x_{\text{tH}} = m_t^2/m_{H^-}^2$ where m_{H^-} is the mass of the charged Higgs. The determination of m_{H^-}

is quite sensitive to changes in the running of the supersymmetric spectrum [40] and thus any change in the running of these parameters will affect the final value of m_{H^-}:

$$m_{H^-}^2 = m_A^2 + M_W^2 + \Delta_H^2, \quad \Delta_H^2 = \frac{-3g_2^2}{32\pi^2} \frac{m_t^4 \mu^2}{\sin^4 \beta M_W^2} \frac{f(M_{\tilde{u}_1,\tilde{u}_2}^2)}{M_{\tilde{u}_1}^2 - M_{\tilde{u}_2}^2}, \quad (6.14)$$

where m_A is the pseudo-scalar Higgs mass and $f(M_{\tilde{u}_1,\tilde{u}_2}^2)$ is a function of the lightest sups: $\tilde{u}_i$ for $i = 1, 2$. The running of μ is quite sensitive to the changes in the running of the susy spectra and as a result $m_{H^-}^2$ is also sensitive. Thus, when we include the running of the second family, the added small changes to the supersymmetric spectrum alters the final value of m_{H^-}.

We have tested the effect of the gluino contribution by computing the ratio, $\delta_{\tilde{g}} = \frac{\hat{K}_{7\mathrm{eff}}^2}{\hat{K}_{7\mathrm{eff}\tilde{g}}^2}$, where these quantities have been defined in eq. (A.8) and eq. (A.16) in appendix A. The ratio is very close to 1 for $m_{1/2} > 300\,\mathrm{GeV}$. At lower gaugino masses, the contribution from gluinos increases but the ratio only decreases by about 1% at $m_{1/2} = 160\,\mathrm{GeV}$. When $m_{1/2}$ is below $300\,\mathrm{GeV}$, the stops become quite light and a NLO calculation is not possible using the approximation of [17]. At lower $m_{1/2}$, a LO calculation has to be used also for charginos, charged Higgs and neutralinos. In fact when $m_{1/2} \leq 280\,\mathrm{GeV}$ this is the case. Thus all the results for $b \to s\,\gamma$ constrained to $m_{1/2} \leq 280\,\mathrm{GeV}$ are calculated at LO.

6.4 Constraints on the parameter ϵ_S

As we have pointed out in section 4, once a family symmetry is introduced, additional contributions to the trilinear couplings are given by the derivatives of the Yukawa couplings with respect to the flavon fields, $\sim Y_{ij}^f F^\alpha \partial_\alpha$, and then in a realistic flavour symmetry $(a_f)_{ij}$ must be of the form of eq. (4.5) with $k_{ij}^f \neq 0$. Given the limitation on the number of parameters that we can use to scan the possibilities of the general form of eq. (5.7), with coefficients $x_{ij}^f \neq 1$ and $z_{ij}^f \neq c_{ij}^f$, we choose to analyze the matrices of the form given in eq. (5.8), and hence effectively allowing $\epsilon_{Sf} > \epsilon_f$. To remain consistent with the approximations made earlier, we maintain the restriction $\epsilon_{Su} < \epsilon_d$.

To this end, we first check once more the running of the off-diagonal squared masses and trilinear couplings with respect to the parameters $\epsilon_{Sf} > \epsilon_f$. For the case of $\epsilon_{Sf} = \epsilon_f$ we have seen in the discussion of section 6.1 (figure 1) that $(M_{\tilde{L}}^2)_{23}$ and $(M_{\tilde{e}}^2)_{23}$ are more sensitive than the quark sector, other than $(M_{\tilde{d}}^2)_{23}$. To be definite, we make the following choices for ϵ_{Sf}:

$$
\begin{aligned}
I. &\qquad \epsilon_{Sd} = \epsilon_{Se} = 0.45, &\qquad \epsilon_{Su} = 0.06, \\
II. &\qquad \epsilon_{Sd} = 0.45, &\qquad \epsilon_{Se} = 0.225, &\qquad \epsilon_{Su} = 0.06, \qquad (6.15)
\end{aligned}
$$

for both cases we keep $c_{22}^d = 0.7$ and $\epsilon_d = 0.1$, $\epsilon_u = 0.05$. In figure 9, we plot then the running of $M_{\tilde{Q}\,23}^2$, $M_{\tilde{u}\,23}^2$, $M_{\tilde{d}\,23}^2$ and $M_{\tilde{L}\,23}^2$ for the two cases of eq. (6.15). These figures can be compared directly with those in figure 1. The only difference being the increased values of the expansion parameters which are now distinct from those in the Yukawa sector.

The running of $M_{\tilde{Q}\,23}^2$ is significantly affected by the increase in ϵ_{Sd} as one can see from the comparison between figures 1a and 9a. This is largely due to the increased importance
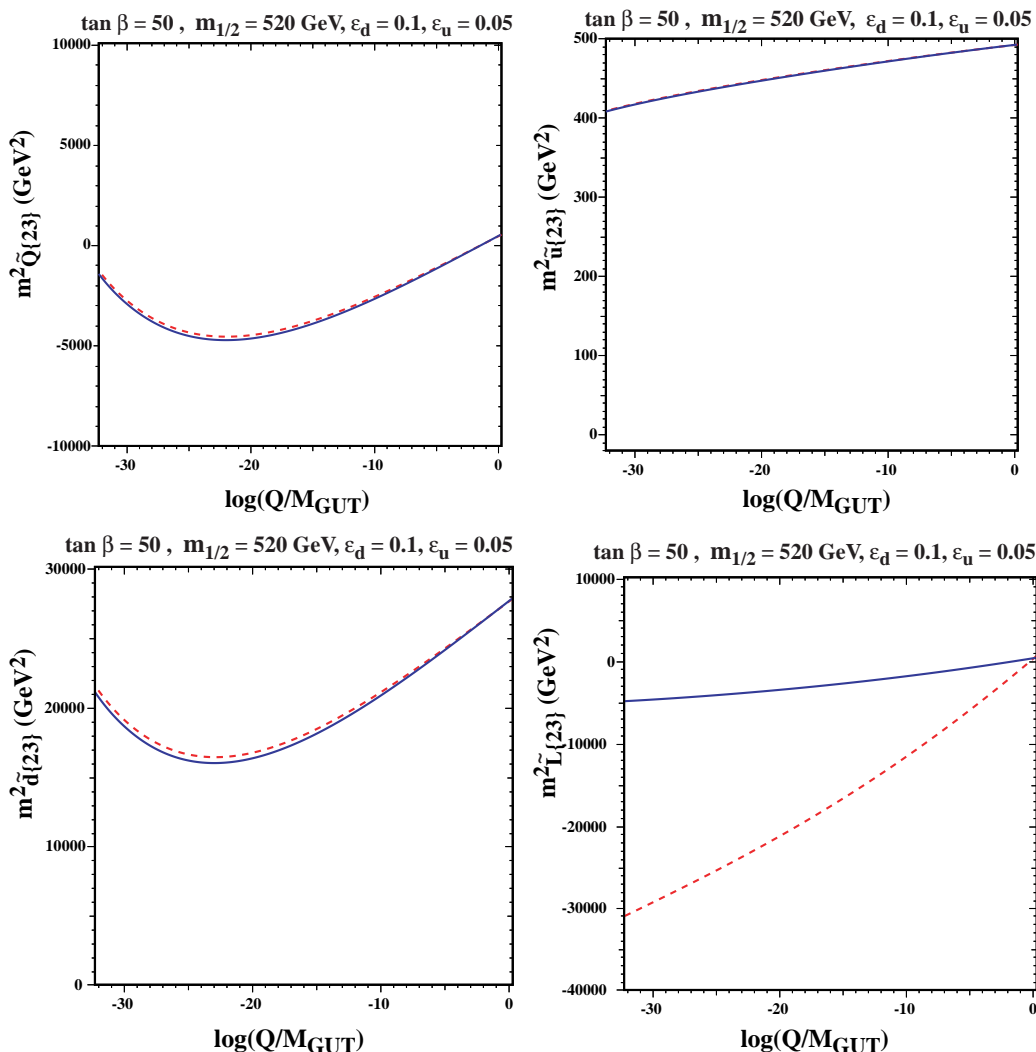
Figure 9: As in figure 1 for case I with $\epsilon_{\text{Sd}} = \epsilon_{\text{Se}} = 0.45$, $\epsilon_{\text{Su}} = 0.06$ (red dashed lines), and for case II with $\epsilon_{\text{Sd}} = 0.45$, $\epsilon_{\text{Se}} = 0.225$ and $\epsilon_{\text{Su}} = 0.06$. (blue solid lines). Here $m_0 = 370\,\text{GeV}$ and $A_0 = -m_{1/2}$.

of the $a_d^\dagger a_d$ term in the beta function which is now competing with the Yukawa terms in eq. (6.1). Not surprisingly the running of $M_{\tilde{u}\ 23}^2$ is quite close to that of figure 1b because we have kept $\epsilon_{\text{Su}} = 0.06$ quite close to $\epsilon_{\text{Su}} = 0.05$. In order to affect the branching ratio for $b \to s\,\gamma$, we are most interested in the effect on the d sector. The initial value of $M_{\tilde{d}\ 23}^2$ is determined by ϵ_{Sd} (and m_0) and is increased relative to the case studied in figure 1c by the the ratio $\epsilon_{\text{Sd}}/\epsilon_d = 0.45^2/0.1^2 = 20.25$. As a result, we can see in figure 9c the enhanced running of $M_{\tilde{d}\ 23}^2$. In panel d), we see the differences between cases I and II, as the running of $M_{\tilde{L}\ 23}^2$ is clearly sensitive to ϵ_{Se}. In figure 10, we show the corresponding effect on the trilinear couplings which can be compared directly with figure 2.

In figure 11, we show the effect of the increased ϵ_S on the parameters $(\delta_{f,XY})_{ij}$ defined
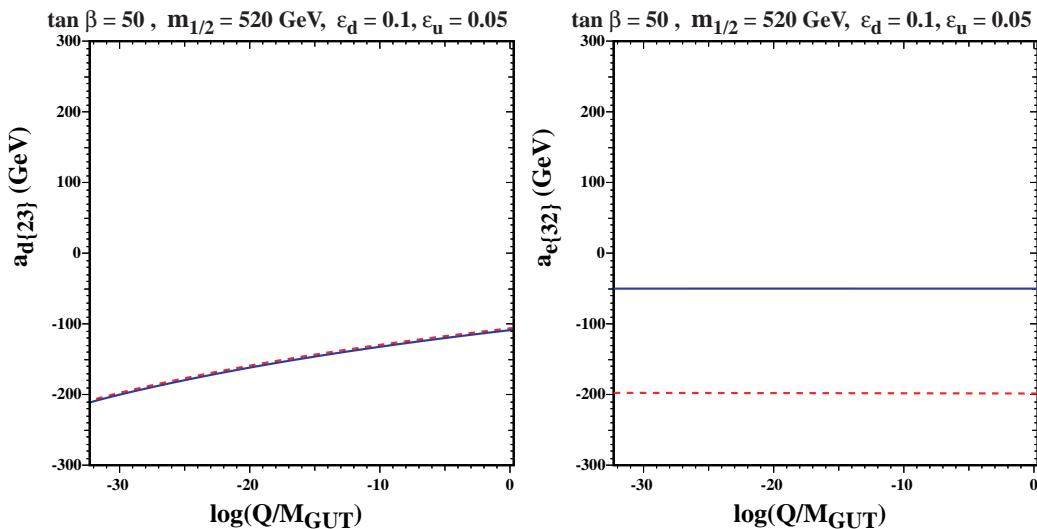
Figure 10: As in figure 9 for the running of a) a_{d23} and b) a_{e32}.

$\|(\delta_{f,XY})_{23}\|$ for $(m_{1/2}, m_0, A_0) = (520, 370, -520)$ GeV			
	I	II	
$\|(\delta_{L,LL})_{23}\|$	0.0012	0.15	0.014
$\|(\delta_{e,RR})_{23}\|$	0.0069	0.50	0.05
$\|(\delta_{e,LR})_{23,32}\|$	6.0×10^{-6}	8.6×10^{-3}	1.5×10^{-3}
$\|(\delta_{e,RL})_{23,32}\|$	4.4×10^{-6}	7.2×10^{-3}	1.1×10^{-3}
$\|(\delta_{Q,LL})_{23}\|$	0.0069	0.009	0.009
$\|(\delta_{d,RR})_{23}\|$	9×10^{-4}	0.021	0.021
$\|(\delta_{d,LR})_{23}\|$	4.3×10^{-6}	8.0×10^{-4}	8.0×10^{-4}

Table 2: Weak scale values for the $\|(\delta_{f,XY})_{23}\|$ (denoted in the 1st column) for $(m_{1/2}, m_0, A_0) = 520, 370, -520)$ GeV. In the 2nd column, we assume $\epsilon_{Sd} = \epsilon_d = 0.1$, $\epsilon_{Su} = \epsilon_u = 0.05$. In column three, we take $\epsilon_{Sd} = 0.45$, $\epsilon_{Su} = 0.06$ and $\epsilon_d = 0.1$, $\epsilon_u = 0.05$ while in column four, we lower ϵ_{Se} to 0.225.

in eq. (6.6). The increased effect on the running of these parameters is also largely due to the increase in the initial values. For example, for $(\delta_{d,RR})_{23}$ the initial value is increased from 9×10^{-3} to 0.021, when $\epsilon_d = \epsilon_{Sd}$ changes from 0.1 to $\epsilon_{Sd} = 0.45$. The comparison to the increase for the other $(\delta_{f,XY})_{23}$ is given in table 2.

If one would like to introduce right handed neutrinos for these cases, then the present analysis will be valid assuming the right handed neutrinos decouple at a higher scale than M_{GUT}, or $M_{\nu_R} > M_{GUT}$. For $M_{\nu_R} < M_{GUT}$ the quark sector will not be largely affected but the lepton sector will be. We will have then important changes to the running of $(M_{\tilde{L}}^2)_{23}$ and $(M_{\tilde{e}}^2)_{23}$. Since the goal in this paper is to study the constraints mainly in the quark sector, the introduction of $M_{\nu_R} < M_{GUT}$ is beyond the scope of the present work. For the present case we consider constraints from leptonic flavour violating process such
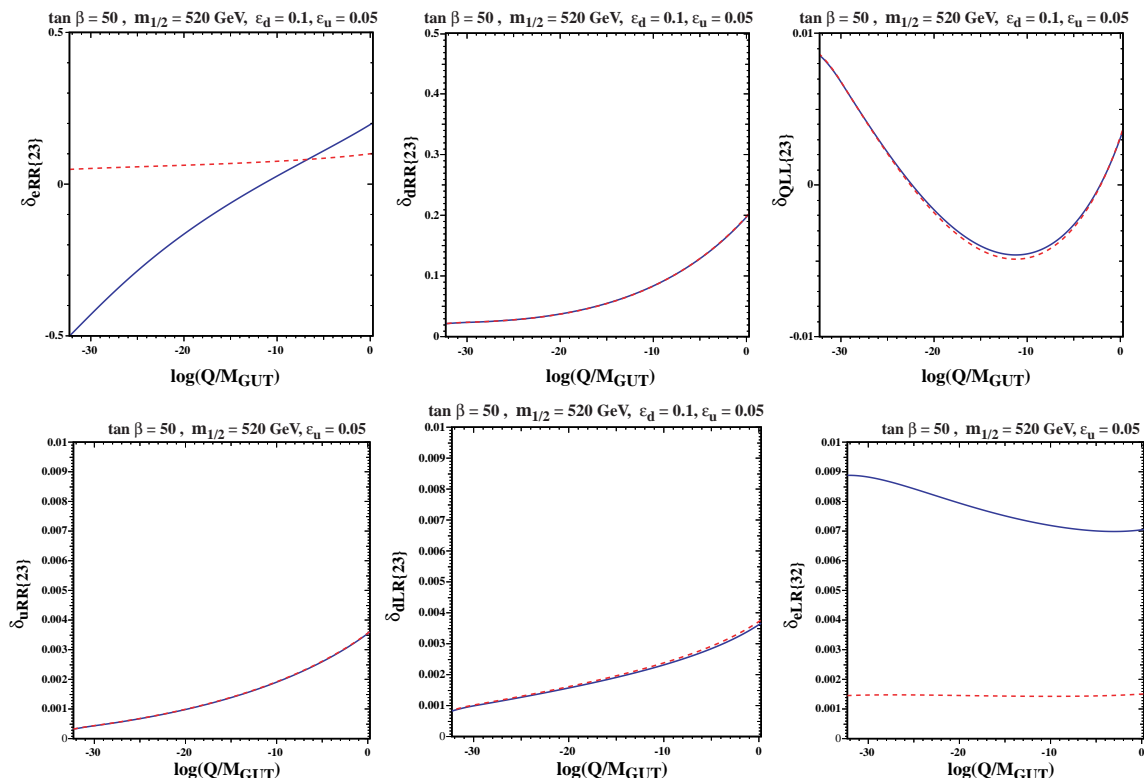
Figure 11: As in figure 9 for the running of $(\delta_{f,XY})_{ij}$ with $f, XY =$ a) e, RR b) d, RR, c) Q, LL, d) u, RR, e) d, LR, f) e, LR.

as $\tau \to \mu\gamma$. For example in the CMSSM, the bounds in terms of the flavour violating parameters eq. (6.6), for $\tan\beta = 50$, are [41]

$$|(\delta_{L,LL})_{23}| \lesssim 0.1, \quad |(\delta_{e,RR})_{23}| \lesssim 0.1, \quad |(\delta_{e,LR})_{23,32}| \lesssim 0.02, \tag{6.16}$$

using the most stringent current bounds. For the case $\epsilon_{Sf} = \epsilon_f$ the values obtained for $(\delta_{l,XX})_{23}$ are well below these bounds:

$$|(\delta_{L,LL})_{23}| = 0.0012, \qquad\qquad |(\delta_{e,RR})_{23}| = 0.0069,$$
$$|(\delta_{e,LR})_{23}| = 6.0 \times 10^{-6}, \qquad\qquad |(\delta_{e,RL})_{23}| = 4.4 \times 10^{-6}. \tag{6.17}$$

For case I of eq. (6.15) the bounds of eq. (6.16) are already saturated while for case II, the parameters are still within the experimental limits.

As can be expected, the sensitivity of $b \to s\,\gamma$ on our expansion parameters for the cases $\epsilon_{dS} > \epsilon_d$ is greater than that discussed in the previous section with $\epsilon_{dS} = \epsilon_d$. The value for the branching ratio for $b \to s\,\gamma$ as a function of $m_{1/2}$ is shown in figure 12. The lower two curves correspond to the single family case (where all mixings are ignored) and our nominal choice of $\epsilon_{Sd} = \epsilon_d$ and are taken directly from figure 8. The upper two curves show the effect of increasing ϵ_{Sd} and ϵ_{Se} using values from case I (upper solid red curve) and case II (green dot-dashed curve). Here we see more clearly that in regions with
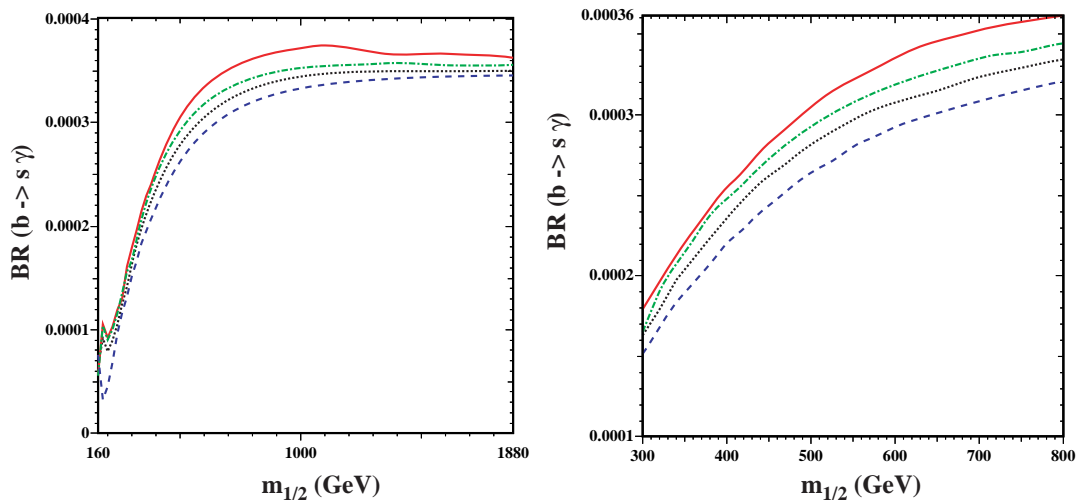
Figure 12: As in figure 8. The curves from bottom to top are as follows: the single family limit is shown by the blue dashed curve; the choice of $\epsilon_d = \epsilon_{\rm Sd} = 0.1$ is shown by the black dotted curve; increased mixing with $\epsilon_{\rm Sd} = 0.45$, $\epsilon_{\rm Se} = 0.225$ is shown by the green dot-dashed curve; and $\epsilon_{\rm Sd} = \epsilon_{\rm Se} = 0.45$ is given by the red solid upper line.

relatively low $m_{1/2}$, the effect of mixings relaxes the constraint imposed by $b \to s\,\gamma$. In these cases, the gluino contributions can be up to 10% for $m_{1/2}$ around 200 GeV but when $m_{1/2}$ increases these contributions reduce considerably. Here the contribution from gluinos does not vary too much from cases I and and II of eq. (6.15), as the running in the $\tilde{d}$ sector is not greatly affected.

To better appreciate the effect of the mixing in the squark sector, we zoom in on the region between $m_{1/2} = 300$ and 800 GeV as shown in panel b) of figure 12. For $\tan\beta = 50$, this region is preferred when other observables such as $(g-2)_\mu$ are included [33, 42]. When theoretical and calculational errors are included, the 95% exclusion value for BR$(b \to s\,\gamma)$ occurs at about 1.9×10^{-4}, as discussed in the previous section and corresponds to a lower limit of 400 GeV on $m_{1/2}$ in the single family case most commonly treated in the CMSSM. This limit is substantially relaxed when two-family mixing is included. For $\epsilon_{\rm Se} = \epsilon_{\rm Sd} = 0.45$ as shown by the upper curve in the figure, we see that the limit is relaxed to $m_{1/2} \gtrsim 335$ GeV. One should keep in mind that the exact numerical limit found here and in the CMSSM is subject to change when NNLO SM corrections are included. On the other hand, we expect the relative change in the limit when two-family mixing is included to be a robust result.

Finally, we show in figure 13 the behaviour of BR$(b \to s\,\gamma)$ on $\epsilon_{\rm Sd}$ for several choices of $m_{1/2}$. As one can see, for low $m_{1/2}$, the branching ratio is lower than the experimental value when $\epsilon_{\rm Sd}$ is small. When $\epsilon_{\rm Sd} \gtrsim 0.4$, the branching ratio increases and the constraint on $m_{1/2}$ is relaxed.

6.5 Sensitivity on the determination of the MSSM spectra

In this last section, we would like to consider the effects of the our flavour violating parameters on the resulting supersymmetric spectrum. Since the parameters ϵ_u, ϵ_d, $\epsilon_{\rm Su}$, $\epsilon_{\rm Se}$, $\epsilon_{\rm Sd}$
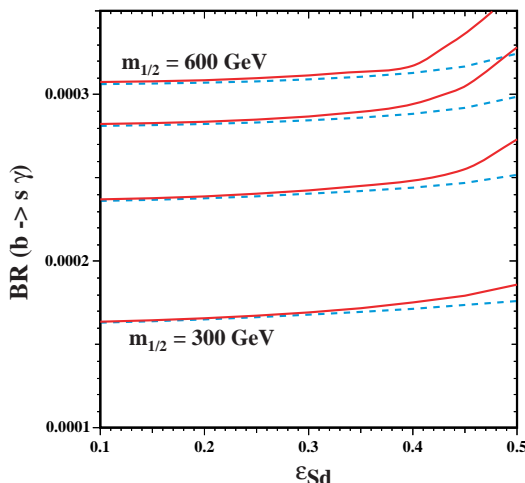
Figure 13: Variation of $BR(b \rightarrow s\,\gamma)$ with respect to ϵ_{Sd}. From top to bottom the lines correspond to $(m_{1/2}, m_0, A_0) = (600, 410, -600)\,\text{GeV}$, $(500, 360, -500)\,\text{GeV}$, $(400, 340, -400)\,\text{GeV}$ and $(300, 380, -300)\,\text{GeV}$.

are small (< 1) and their dependence on the Yukawa, trilinear and soft squared matrices is quadratic, eqs. (5.7), (5.8), we do not expect a big effect on the spectra, except for sensitive parameters such as

$$\mu, \; m_{H_{1,2}}, \; m_A, \tag{6.18}$$

which are determined from the electroweak symmetry breaking conditions. In figure 14, we show the dependence of the six masses of the sdown sector and of the charged slepton sector with respect to ϵ_{Sd} for the case

$$\epsilon_{Se} = \frac{1}{2}\epsilon_{Sd}, \tag{6.19}$$

with $\epsilon_{Su} = 0.06$ which includes case II of eq. (6.15) for $\epsilon_{Sd} = 0.45$. Here we have fixed $m_{1/2} = -A_0 = 520\,\text{GeV}$, $m_0 = 370\,\text{GeV}$, and $\tan\beta = 50$. As one can see, only when $\epsilon_{Sd} \gtrsim 0.35$, does the splitting in the masses of the heaviest families (1st two generations) become relevant. The third generation (mostly sbottom) masses are relatively insensitive to ϵ_{Sd} and are close to their CMSSM (with no flavour violation) values $(M_{\tilde{d}_1}, M_{\tilde{d}_2}) = (956, 1017)\,\text{GeV}$ as shown by the lower two (red) curves in figure 14a. where $1,2,\ldots 5,6$ are ordered lightest to heaviest. Analogously for the 3rd generation sleptons we have $(M_{\tilde{e}_1}, M_{\tilde{e}_4}) = (246, 477)\,\text{GeV}$, in the CMSSM as shown by the two (red) curves in panel b. Note that the heavier stau is heavier than the mostly right-handed selectron and smu. With no flavour violation, the masses of the 1st two generations are degenerate (due to the smallness of the associated fermion masses) in the CMSSM. Here we see the degree of splitting induced when $\epsilon_{Sd} \neq 0$. We do not show the dependence of the of the sups with respect to ϵ_{Sd} or ϵ_{Su} because their change (with respect the running of just the third family) is less than the 0.4%. We note, however, a more important difference in the determination
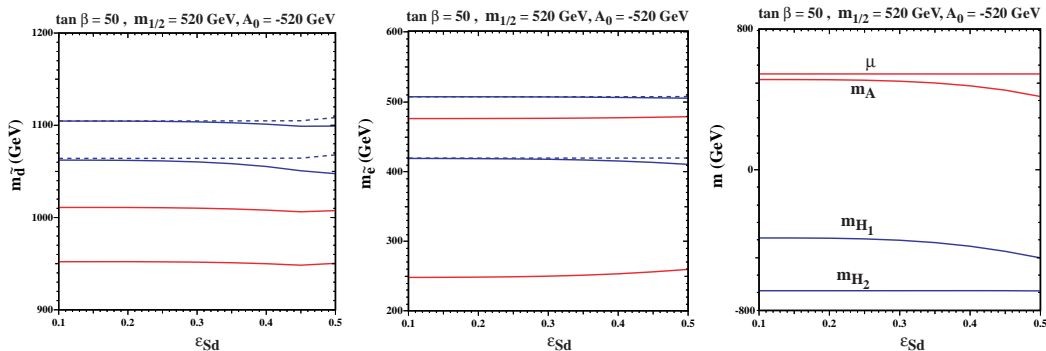
– 29 –

Figure 14: Masses for the sdowns (a), and charged sleptons (b) sectors as a function of ϵ_{Sd} for the case $(m_{1/2}, m_0, A_0) = (520, 370, -520)\,\text{GeV}$, $\epsilon_{\text{Se}} = \frac{\epsilon_{\text{Sd}}}{2}$, $\epsilon_d = 0.1$, and $\epsilon_u = 0.05$. In panel c, the soft Higgs masses, μ and m_A are shown as labeled.

of the running Higgs sector parameters which is displayed in panel c of figure 14. When $\epsilon_{\text{Sd}} = 0$, we have $(\mu, m_{H_1}, m_{H_2}, m_A) = (568, -403, -708, 516)\,\text{GeV}$ at the electroweak scale, where m_{H_i} are the two soft Higgs masses, and the minus refers to the sign of the squared mass. For $\epsilon_{\text{Sd}} = 0.45$ these become $(\mu, m_{H_1}, m_{H_2}, m_A) = (545, -465, -688, 452)\,\text{GeV}$. Since the most important supersymmetric contribution to $\text{BR}(B_s \to \mu^+\mu^-)$ scales as $f_{B_s}^2/m_A^4$ and in the analysis that we have done the inclusion of the second family tends to lower m_A, we expect that in general $\text{BR}(B_s \to \mu^+\mu^-)$ increases from the case $\epsilon_{\text{Sd}} = 0$ to $\epsilon_{\text{Sd}} = 0.45$, but remains within the experimental limit.

7. Summary

Flavour symmetries (FS) are often constructed using supersymmetry. Departures from MFV will affect the supersymmetric spectra and constraints at the electroweak scale can be employed to determine the profile of the Yukawa matrices at the GUT scale. Flavour violating processes then can be used to further constrain the shape of such Yukawa matrices and the parameters associated with the FS.

Clearly, a completely general analysis is not practical for computational purposes, as it involves many new supersymmetric parameters in addition to the Yukawa matrices. To constrain the parameters of the latter we can rely on the determination of the Yukawa matrices at M_{EW} from flavour violating parameters in the SM, assuming a hierarchical symmetric form, eq. (3.1) (although in this context a non-symmetric form is equally plausible, it requires more parameters) such that the expansion parameter in the up sector is smaller than in the down sector, motivated by the different hierarchies between up and down quarks respectively, $\epsilon_u \sim \epsilon_d^2$. For the latter case we can simplify the usual relations obtained in FS between the expansion parameters of the Yukawa matrices and the off-diagonal parameters of the trilinear couplings and soft squared mass matrices at the GUT scale.

In our analysis, we used an iterative procedure as is common in the analysis of the CMSSM [28, 29], with the inclusion of three more parameters describing the Yukawa matrices $(\epsilon_u, \epsilon_d$ and $c_{22}^d)$, at the GUT scale. Other necessary inputs are the values of $m_s(2\text{GeV})$,

$m_c(2\mathrm{GeV})$, $m_\mu(M_{\mathrm{EW}})$, and the values of the CKM matrix relevant to the two heaviest families. These are used to determine seven more parameters in the Yukawa sector at the GUT scale, as in eq. (5.2).

The important results of this analysis in the fermion sector are as follows. First, using the supersymmetric corrections to the fermion masses of the two heaviest families, at the EW scale, we can accurately determine the profile of the Yukawa matrices at the GUT scale. Particularly important is the ratio $(Y_d)_{23}(M_{\mathrm{GUT}})/(Y_d)_{22}(M_{\mathrm{GUT}})$. Whereas in an analysis where the running of supersymmetric parameters is neglected, this ratio is found to be 1.3, when supersymmetric corrections are taken into account, this ratio is enhanced to 1.8, cf., eq. (6.12). Second, the value of the expansion parameter, ϵ_d, is effectively constrained by the appropriate determination of $m_s(M_{\mathrm{EW}})$, with its corresponding supersymmetric corrections. Of course the exact value depends on the values for the coefficients used in the expression of the Yukawa matrices, for example $(Y_d)_{22} = c_{22}^d \epsilon_d^2$. Assuming that the coefficients c_{ij}^d should be of order one, we find for example that for $c_{22}^d = 0.7$, $\epsilon_d \in [0.085, 0.10]$, otherwise m_s is predicted to be either too small or too big, respectively. This determination is important when constructing FS. The exact values of $(Y_f)_{ij}(M_{\mathrm{GUT}})$ are used to justify different choices of family groups (e.g. SU(3), SO(3)). We have only considered the running of the two heaviest families and hence the structure of the lightest family cannot be determined at the GUT scale by this analysis. However, the inclusion of the lightest family will not have a strong impact on the results obtained here.

Our results are also relevant for the supersymmetric sector. When off-diagonal terms are introduced in the sfermion mass matrices or trilinear terms at the GUT scale, departures from MFV are induced at the EW scale. In the most constrained models, ie., when no new parameters are introduced as in eq. (5.7), this departure is small and the resulting sensitivity of BR(b $\rightarrow$ s γ) is not significant. In contrast, when the expansion parameters, ϵ_{Sf}, for the supersymmetric sector differ from that of the fermion sector, ϵ_f, we may obtain more sizable departures from MFV. We have compared the running of the off-diagonal trilinear couplings and soft squared mass matrices for these two possibilities.

Our calculations were performed using the values of the supersymmetric parameters as obtained when running the full MSSM, with the simplified structure for the trilinear couplings and soft squared masses at two loops as described above. Hence we do not rely on estimating the value of the flavour violating parameters $(\delta_{f_{\mathrm{XY}}})$, eq. (6.6), at the EW scale. Indeed, these are calculated directly in terms of the parameters introduced at the GUT scale as an extension of the CMSSM. These parameters were introduced in section 6.1 to emphasize their running from M_{GUT} to M_{EW}. Current limits on the charged slepton sector could be used to further constrain the choice of the expansion parameters ϵ_f and ϵ_{Sf}.

We have seen that the running of $(\delta_{f_{\mathrm{XY}}})_{23}$ is quite limited when initial conditions at the GUT scale are constrained. This is true for both the case of MFV at the GUT scale as well as the case when the expansion parameters of the supersymmetric sector differ from those of the fermion sector, $\epsilon_{\mathrm{Sf}} \neq \epsilon_f$, as we saw from table 2. In column 2 of that table, we have assumed MFV at the GUT scale and the departures from MFV at the weak scale are small as seen by the small values of the δ-parameters. In this case, the bounds on the

parameters $(\delta_{(e,L)_{\mathrm{XY}}})_{23}$ are well below the bounds imposed by $\mathrm{BR}(\tau \to \mu\gamma)$ in eq. (6.16). However, if we take for example $\epsilon_{\mathrm{Sd}} = \epsilon_{\mathrm{Se}} = 0.45$, we can see that the values obtained for $(\delta_{(e,L)_{\mathrm{XY}}})_{23}$ in column 3 already saturate the bounds imposed by $\mathrm{BR}(\tau \to \mu\gamma)$. In the last column, we showed the δ's when $\epsilon_{\mathrm{Sd}} = 0.45$ and $\epsilon_{\mathrm{Se}} = 0.225$ where the bounds imposed by $\mathrm{BR}(\tau \to \mu\gamma)$ are satisfied.

We have linked the down-type quarks to the charged leptons by generating the off-diagonal terms of the Yukawa matrices of both with V_{CKM} at EW scale as in eq. (5.3). This is motivated by GUT theories where often charged leptons and down-type quarks acquire Yukawa matrices of the same form. Extending the relation to the supersymmetric sector, by taking the expansion parameters ϵ_{Sd} and ϵ_{Se} to be of the same order, we naturally find that the parameters $|(\delta_{(Q,d)XY})_{23}|$ and $|(\delta_{(L,e)XY})_{23}|$ are correlated at EW scale. From table 2, we see that for $\epsilon_{\mathrm{Sd}} = \epsilon_{\mathrm{Se}}$ we have in fact $|(\delta_{(Q,d)XX})_{23}| = O\left(|(\delta_{(L,e)XX})_{23}|\right)$. Applying the bounds on $|(\delta_{(L,e)XX})_{23}|$ imposed by $\mathrm{BR}(\tau \to \mu\gamma)$, we have taken $\epsilon_{\mathrm{Se}} = 0.5\epsilon_{\mathrm{Sd}} = 0.225$ and this effectively decreases $|(\delta_{(L,e)XX})_{23}|$ by an order of magnitude with respect to $|(\delta_{(Q,d)XX})_{23}|$.

Finally, we have calculated $\mathrm{BR}(\mathrm{b} \to \mathrm{s}\,\gamma)$ at NLO for the charged Higgs, chargino and neutralino contributions and at LO for the gluino contributions (since a NLO computation is not yet fully available for this case). BLO corrections for the gluino contributions were considered in [19], but given the values of $(\delta_{f,XY})_{23}$ that we have derived, these corrections will have little impact on the value $\mathrm{BR}(\mathrm{b} \to \mathrm{s}\,\gamma)$ that we obtain.

Given the SM NNLO computation and the continuously improved experimental value, it is important to determine $\mathrm{BR}(\mathrm{b} \to \mathrm{s}\,\gamma)$ as precise as possible in a given supersymmetric model. We have seen that the structures for Yukawa matrices, trilinear terms and soft-matrices of the form of eqs. (5.7), (5.8) yield a positive contribution to the calculation of $\mathrm{BR}(\mathrm{b} \to \mathrm{s}\,\gamma)$ for $\mu > 0$ bringing the branching ratio closer to its experimental value than what would have been obtained in the absence of any FV. In fact when $\epsilon_{\mathrm{Sf}} \neq \epsilon_f$, such that $\epsilon_{\mathrm{Sd}} > \epsilon_d > \epsilon_{\mathrm{Su}} > \epsilon_u$, it is possible to relax the lower limit of $\sim 400\,\mathrm{GeV}$ on $m_{1/2}$ found in the single family case most commonly treated in the CMSSM. For $\epsilon_{\mathrm{Se}} = \epsilon_{\mathrm{Sd}} = 0.45$ this limit is relaxed to $m_{1/2} \gtrsim 335\,\mathrm{GeV}$, as we have discussed in section 6.4.

A. $\mathrm{b} \to \mathrm{s}\gamma$ beyond minimal flavour violation

At LO the renormalization evolution of the gluino contribution to the effective Hamiltonian, $\mathcal{H}_{\mathrm{eff}}$, from M_{EW} down to μ_b is different than the contribution from W^-, H^-, charginos and neutralinos [14]. Hence, $\mathcal{H}_{\mathrm{eff}}$ can be written as:

$$\mathcal{H}_{\mathrm{eff}} = \mathcal{H}_{\mathrm{eff}}^{W^-,\,H^-,\,\chi} + \mathcal{H}_{\mathrm{eff}}^{\tilde{g}}. \tag{A.1}$$

"Mixed" diagrams containing the W boson, gluinos and squarks give rise to α_s corrections to the Wilson coefficients in $\mathcal{H}^{W^-,\,H^-,\,\chi}$ at the matching scale, and they are taken into account at the NLO level. We include the contributions of gluinos at LO and those from H^-, charginos and neutralinos at NLO.

We recall that the starting point in the calculation of inclusive B decay rates is the low-energy effective Hamiltonian [20]

$$\mathcal{H}_{\text{eff}}^{W^-,\,H^-,\,\chi} = -\frac{4G_F}{\sqrt{2}}\, V_{\text{ts}}^* V_{\text{tb}} \sum_i C_i(\mu_b) O_i(\mu_b)\,. \qquad (A.2)$$

In the massless strange quark limit, the operators relevant to our discussion are

$$
\begin{aligned}
O_2 &= \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L\,,\\
O_7 &= \frac{e\, m_b}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R\,,\\
O_8 &= \frac{g_s m_b}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} G_a^{\mu\nu} t_a b_R\,.
\end{aligned}
\qquad (A.3)
$$

To an excellent approximation, the contributions of other operators can be neglected. The renormalization scale μ_b in (A.2) is conveniently chosen of order m_b, so that all large logarithms reside in the Wilson coefficient functions. However when the mass of the strange quark is taken into account we need to consider the operators

$$
\begin{aligned}
O_7' &= \frac{e\, m_b}{16\pi^2}\, \bar{s}_R \sigma_{\mu\nu} F^{\mu\nu} b_L\,,\\
O_8' &= \frac{g_s m_b}{16\pi^2}\, \bar{s}_R \sigma_{\mu\nu} G_a^{\mu\nu} t_a b_L\,.
\end{aligned}
\qquad (A.4)
$$

For these operators the corresponding Wilson coefficients $\tilde{C}^{H^-} \propto m_s m_b / m_t^2 \tan^2\beta$ and $\tilde{C}^{\chi^\pm}$ are quite small. For $\mathcal{H}_{\text{eff}}^{\tilde{g}}$ we have

$$\mathcal{H}_{\text{eff}}^{\tilde{g}} = \sum_i C_{i,\tilde{g}}(\mu_b) O_{i\tilde{g}}(\mu_b) + \sum_i C_{i,\tilde{g}}'(\mu_b) O_{i\tilde{g}}'(\mu_b) + \sum_i \sum_q C_{i,\tilde{g}}^q(\mu_b) O_{i\tilde{g}}^q(\mu_b), \qquad (A.5)$$

where q runs over the light quarks $q = u, d, c, s, b$. The operators $O_{i,\tilde{g}}(\mu_b)$ can be of two types, one for $O_{j\ b,\tilde{g}}(\mu_b)$ and the other for $O_{j\ \tilde{g},\tilde{g}}(\mu_b)$, $j = 7, 8$. The former can be found from eq. (A.3) with the substitution of $(1/16\pi^2)$ with g_s^2 and the latter with the substitution of $(m_b/16\pi^2)$ with g_s^2. The primed operators in eq. (A.5) come from the interchange of $L \leftrightarrow R$. The operators $O_{i\tilde{g}}^q(\mu_b)$ are suppressed with respect to the magnetic operators by the mass of the corresponding quark or by an extra power of g_s. We neglect them here since their contribution is quite tiny with respect of the operators in the first two terms of eq. (A.5).

In the SM, the complete theoretical prediction for the $B \to X_s \gamma$ decay rate at NLO was presented for the first time by Chetyrkin et al. [21]. It depends on a parameter δ defined by the condition that the photon energy be above a threshold given by $E_\gamma > (1-\delta)E_\gamma^{\max}$, where $E_\gamma^{\max} = m_b/2$ is the maximum photon energy attainable in the parton model. Thus, $\delta = 1 - 2E_\gamma^{\max}/m_b$, and we use $E_\gamma^{\max} = 1.6\,\text{GeV}$, as it is the value at which the HFAG collaboration has presented its latest results [9, 10].

The prediction for the $B \to X_s \gamma$ branching ratio is usually obtained by normalizing the result for the corresponding decay rate to that for the semileptonic decay rate, thereby eliminating the strong dependence on the b-quark mass:

$$\text{BR}(B \to X_s \gamma)\big|_{>(1-\delta)E_\gamma^{\max}} = \text{BR}(B \to X_c e\bar{\nu})_{\exp} \frac{\Gamma(B \to X_s \gamma)|_{E_\gamma > (1-\delta)E_\gamma^{\max}}}{\Gamma(B \to X_c e\bar{\nu})}. \qquad (A.6)$$

Following closely the formalism of [22, 23], we write

$$\text{BR}(B \to X_s \gamma)\big|_{E_\gamma > E_0} = \text{BR}(B \to X_c e \bar{\nu})_{\text{exp}} \left| \frac{V_{\text{ts}}^* V_{\text{tb}}}{V_{\text{cb}}} \right|^2 \frac{6\alpha_{\text{em}}}{\pi} [P(E_0) + N(E_0)] \frac{1}{r(\Gamma_u/\Gamma_c)} \quad \text{(A.7)}$$

where perturbative, $P(E_0)$, and non-perturbative, $N(E_0)$, contributions are treated separately, and $E_0 = (1-\delta)E_\gamma^{\text{max}}$. The function $N(E_0)$ is a correction which does not depend on the renormalization of the running mass m_b from μ_{EW} to m_b, it contains power corrections that arise from the interference between the current-current operator and the magnetic dipole operator, hence called the "non-perturbative" contribution. The precise definition is given in eq. (3.10) of ref. [22]. In the SM this is [22], $N(E_0 = 1.6 \text{ GeV}) = 0.0036 \pm 0.0006$. The ratio $r(\Gamma_u/\Gamma_c)$ is a semileptonic phase space factor which is independent of the calculational problem of convergence of the perturbation series in $b \to X_s \gamma$. This has been calculated accurately [22] up to NNLO in the SM.

Since $\text{BR}(B \to X_c e \bar{\nu})$ is an experimental input, only the function $P(E_0)$ has contributions from processes beyond the SM. Using the approach of Gambino et. al. [22], rather than writing the terms $P(E_0) + N(E_0)$ as functions of the standard Wilson coefficients, eq. (A.2), at the decay scale, μ_b, these functions can be written in terms of functions involving the renormalization effects from μ_0 (m_t or μ_{EW}) to μ_b and the Wilson coefficients at μ_0. The contributions from the charm quark do not get re-normalized from M_{EW} to m_b, while the contributions from t do. Hence instead of the familiar contribution of $|C_7^{\text{eff}}(\mu_{\text{EW}})|^2$ to $\text{BR}(B \to X_s \gamma)$, we have

$$|C_7^{\text{eff}}(\mu_b)|^2 \to \hat{K}_{7\text{eff}}^2 \equiv P(E_0) + N(E_0)$$
$$P(E_0) = |X_c + X_t + \epsilon_{\text{EW}}|^2 + B(E_0), \quad \text{(A.8)}$$

where X_c and X_t represent the c and t contributions respectively, ϵ_{EW} is an electroweak correction and $B(E_0)$ is the bremsstrahlung correction from $b \to s\gamma g$ and $b \to s\gamma q\bar{q}$ with $q = u, d, s$ [23].

Analogous to the usual Wilson coefficients, we can decompose

$$X_f(\mu_b) = X_f^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} X_f^{(1)}(\mu_b), \ f = c, t. \quad \text{(A.9)}$$

The effects of physics beyond the SM can be incorporated [23] as:

$$C_i^{(0)\text{TOT}}(\mu_{\text{EW}}) = C_i^{(0)\text{SM}}(\mu_{\text{EW}}) + C_i^{(0)\text{BSM}}(\mu_{\text{EW}}), \ i = 7, 8, \quad \text{(A.10)}$$

where $C_i^{(0)\text{SM}}(\mu_{\text{EW}}) = C^{(0)\text{eff}}(\mu_{\text{EW}})_i$, $i = 7, 8$ correspond to the coefficients of eq. (12) of [21], i.e. the scheme independent Wilson coefficients, the coefficients $C_i^{(0)\text{BSM}}(\mu_{\text{EW}})$ of course refer to processes beyond the SM, in this case the MSSM. From here on, we will refer to the following coefficients $\tilde{C}_7^{(0)}(\mu_{\text{EW}}) = C_7^{(0)\text{TOT}}(\mu_{\text{EW}}) + \frac{23}{36}$ and $\tilde{C}_8^{(0)}(\mu_{\text{EW}}) = C_8^{(0)\text{TOT}}(\mu_{\text{EW}}) + \frac{1}{3}$. For the MSSM, the general form for $C_i^{(0)\text{MSSM}}(\mu_{\text{EW}})$ is known, although the contribution from gluinos does not follow the same pattern of renormalization from M_{EW} to m_b and it should be implemented in a different fashion. The contribution

from charginos, neutralinos and charged Higgs bosons can be renormalized as in the SM, although the complete NLO corrections to $C_{i=7,8}^{\text{MSSM}}(\mu_b)$ are not known.

Thus, if we have the following hierarchy of masses

$$\mu_{\text{SUSY}} \sim O(m_{\tilde{g}}, m_{\tilde{q}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}) \gg \mu_{\text{EW}} \sim O(M_{\text{EW}}, m_t, m_{H^\pm}) \gg \mu_b, \qquad (A.11)$$

we expect that the following assumptions represent a first good approximation for taking into account the contribution from charginos, neutralinos and charged Higgs bosons:

$$\tilde{C}_i^{(0)} = \tilde{C}_i^{(0)SM} + C_i^{(0)\chi,H^\pm}, \ i = 7, 8. \qquad (A.12)$$

We note that $C_7^{(0)\tilde{g}}(\mu_b)$ must be added on directly to $X_t^{(0)}(\mu_b)$ because gluinos undergo a different renormalization from M_{EW} to μ_b. The expressions that we use for $C_{7,8}^{(0)\chi,H^\pm}$ and $C_{\tilde{g}}^{(0)}$ are given in eqs. A16-A20 of [24]. At NLO for $\tilde{C}_{7,8}^{XH^\pm}$, we follow the approach of [17].

Thus at LO, eq. (A.8) would receive the following contributions from gluinos at μ_b:

$$P(E_0)_{\tilde{g}} = \left| X_{\tilde{g}}^{(0)}(\mu_b) + X_c(\mu_b) + X_t(\mu_b) + \epsilon_{\text{EW}} \right|^2 + \left| X_{\tilde{g}}^{(0)\prime}(\mu_b) \right|^2 + B(E_0), \quad (A.13)$$

where

$$X_{\tilde{g}}^{(0)}(\mu_b) = -\frac{16\sqrt{2}\pi^3 \alpha_s(\mu_b)}{G_F V_{\text{tb}} V_{\text{ts}}^*} \left[C_{7b,\tilde{g}}^{(0)}(\mu_b) + \frac{1}{m_b} C_{7\tilde{g},\tilde{g}}^{(0)}(\mu_b) + \frac{m_c}{m_b} C_{7c,\tilde{g}}^{(0)}(\mu_b) \right], \qquad (A.14)$$

and $X_{\tilde{g}}^{(0)\prime}(\mu_b)$ is the analogous contribution from the primed operators (of flipped chirality $L \leftrightarrow R$) of eq. (A.5). The bremsstrahlung function $B(E_0)$, also receives gluino contributions [23], which are added to the Wilson coefficients at μ_b

$$C_i^{(0)\text{TOT}}(\mu_b) = C_i^{(0)SM}(\mu_b) + C^{(0)\chi,H^\pm}(\mu_b) + X_{\tilde{g}}^{(0)}(\mu_b), i = 7, 8, \qquad (A.15)$$

the contribution from BSM processes to the rest of the Wilson coefficients, $i = 1, \ldots, 6$, is negligible. We will denote by $K_{7\text{eff}\tilde{g}}^2$ the contribution to BR(b → s γ) containing the Wilson operators from gluinos:

$$\hat{K}_{7\text{eff}\tilde{g}}^2 \equiv P(E_0)_{\tilde{g}} + N(E_0). \qquad (A.16)$$

As stated before we have implemented the calculation of BR(b → sγ) as introduced in [22], and adopted in [23] for the contribution of beyond SM effects. The main difference in the way BR(b → sγ) is calculated in the SM as introduced in [22], with respect to previous analyses is that $m_c^{\text{pole}}/m_b^{\text{pole}}$ is replaced by $m_c^{\bar{MS}}(m_c)/m_b^{\text{pole}}$ in the element $\langle X_s \gamma | (\bar{s}c)_{V-A}(\bar{c}b)_{V-A} | b \rangle$. In addition, the running of the charm and top contributions from the EW scale to the decay scale μ_b is different. In table 3, we compare the values obtained in our calculation for the different choices of $m_c^{\text{pole}}/m_b^{\text{pole}} = 0.22$ and $m_c^{\bar{MS}}(m_c)/m_b^{\text{pole}} = 0.3003$. Lines 1 to 5 correspond to the calculation of BR(b → s γ) implemented here following [22]. Line 5 is the SM value that we obtain in the decoupling limit. Line 4 is what we obtain when using the top mass assumed in [22], and is in perfect agreement with the value obtained there (eq. (4.14) of that reference). Line 3 corresponds to the SM when $\delta = 0.9$ shown for comparison to other works. Lines 1 and 2 assume $m_c^{\text{pole}}/m_b^{\text{pole}} = 0.22$ for two values of m_t. The remaining parameters for the calculation of BR(b → s γ) that we use and that were not specified before are BR(B → $X_c e\bar{\nu}$)$_{\text{exp}}$ = 0.1059, $\left| \frac{V_{\text{ts}}^* V_{\text{tb}}}{V_{\text{cb}}} \right|^2 = 0.97$ and $\mu_b = 5\,\text{GeV}$.

BR(b → s γ) for the SM			
m_t	E_γ	$\frac{m_c}{m_b}$	BR(b → sγ)$[10^{-4}]$
174.3 GeV	1.6 GeV	0.3003	3.32 ± 0.19
171.4 GeV	1.6 GeV	0.3003	3.27 ± 0.19
171.4 GeV	0.2345 GeV	0.22	3.71 ± 0.20
174.3 GeV	1.6 GeV	0.22	3.61 ± 0.20
171.4 GeV	1.6 GeV	0.22	3.59 ± 0.20

Table 3: Input values for the evaluation of BR(b → sγ). For these cases, we have used $\alpha_s(m_Z) = 0.1185$. The value of $E_\gamma = 0.2345$ GeV corresponds to $\delta = 0.9$. The error quoted is just the parametric error.

B. Interactions of fermion, sfermions and sparticles

B.1 Quarks-squarks-gluino interactions

The interaction between quarks-squarks and gluinos is described by the Lagrangian:

$$-\mathcal{L}_{q-\tilde{q}-\tilde{g}} = \sqrt{2}g_s T^a_{\alpha\beta} \left[-\bar{q}'^\alpha_i \mathcal{P}_L \tilde{g}_a \tilde{q}'^\beta_{Ri} + \bar{q}^\alpha_i \mathcal{P}_R \tilde{g}_a \tilde{q}^\beta_{Li} + h.c \right], \qquad (B.1)$$

in the electroweak-color basis, where $\mathcal{P}_{R,L} = (1 \pm \gamma_5)/2$, a is the gluino color index, α, β are the quark-squark color indices and i is the generation index. In the gauge basis, the couplings at the quark-squark-gluino vertex are given by, for *incoming* gluinos to sfermion-fermion, $C_{\tilde{g}\tilde{f}_{(2l-1)}(f'_L)_l} = -ig_3\sqrt{2}T = -i\frac{2}{\sqrt{3}}g_3$. In this basis the effective mass Lagrangian that we construct from the soft Lagrangian of eq. (2.1) is

$$\mathcal{L}^{\text{eff}}_{m_{\tilde{q}}} = -(\tilde{q}'_L, \tilde{q}'_R)_i (\mathcal{M}^2_{\tilde{q}'})_{ij} \begin{pmatrix} \tilde{q}'^*_L \\ \tilde{q}'^*_R \end{pmatrix}_j, \qquad (B.2)$$

where

$$(\mathcal{M}^2_{\tilde{f}})_{ij} = \begin{bmatrix} M^2_{\text{LL}} & M^{2\dagger}_{\text{LR}} \\ M^2_{\text{LR}} & M^2_{\text{RR}} \end{bmatrix}_{ij}$$

$$= \begin{bmatrix} (M^2_{\tilde{Q}})_{ij} + (M^2_f)_{ij} + D^f_L & -(a_{f_{ij}}v_f + \mu \tan^p \beta (M_f)_{ij}) \\ -\left(a^*_{f_{ij}}v_f + \mu \tan^p \beta (M^*_f)_{ij}\right) & (M^2_{\tilde{f}_R})_{ij} + (M^2_f)_{ij} + D^f_R \end{bmatrix}$$

$$D^f_{L,R} = \cos 2\beta M^2_Z \left(T^3_f - \mathcal{Q}_{f_{L,R}} \sin^2 \theta_W \right), \quad p = \begin{cases} 1, & f = d \\ -1, & f = u. \end{cases} \qquad (B.3)$$

In eq. (B.3) $i, j = 1, 2, 3$ are the family indices, and $(M_f)_{ij}$ are the non-diagonalized fermion mass matrices. We recall that in the eigenmass formalism all the interactions are computed with the particles that are mass eigenstates. Hence the effective soft mass matrix of eq. (B.2), with the corresponding family elements eq. (B.3), needs to be rotated where the fermions are mass-eigenstates, i.e. to the so-called super CKM (SCKM) basis. For the diagonalization of fermions we adopt the convention

$$(u_{L,R})_i = (V^{u\dagger}_{L,R})_{ij}(u'_{L,R})_j, \quad (d_{L,R})_i = (V^{d\dagger}_{L,R})_{ij}(d'_{L,R})_j, \quad \rightarrow \quad V_{\text{CKM}} = V^{u\dagger}_L V^d_L, \quad (B.4)$$

and thus the sfermions must be rotated similarly:

$$
(\tilde{u}_{L,R})_i = \left(V_{L,R}^{u\dagger}\right)_{ij} (\tilde{u}'_{L,R})_j , \quad \left(\tilde{d}_{L,R}\right)_i = \left(V_{L,R}^{d\dagger}\right)_{ij} \left(\tilde{d}'_{L,R}\right)_j . \tag{B.5}
$$

Then the soft mass matrix eq. (B.3) becomes

$$
\left(\mathcal{M}_{\tilde{f}}^{\mathrm{SCKM}}\right)^2_{ij} = \begin{bmatrix} M^{\mathrm{SCKM}2}_{\mathrm{LL}} & M^{\mathrm{SCKM}\dagger 2}_{\mathrm{LR}} \\ M^{\mathrm{SCKM}2}_{\mathrm{LR}} & M^{\mathrm{SCKM}2}_{\mathrm{RR}} \end{bmatrix}_{ij} \equiv \left(\widehat{\mathcal{M}}_{\tilde{f}}^2\right)_{ij}
$$

$$
= \begin{bmatrix} \left(V_L^{\mathrm{fT}} M_{\tilde{Q}}^2 V_L^{f*}\right)_{ij} + \hat{M}_{f_i}^2 + D_L^f & - \left(\left(V_L^{\mathrm{fT}} a_f V_R^{f*}\right)_{ij} v_f + \mu^* \tan^p \beta \hat{M}_{f_i}\right) \\ - \left(\left(V_L^{f\dagger} (a_f^*)_{ij} V_R^f\right) v_f + \mu \tan^p \beta \hat{M}_{f_i}\right) & \left(V_R^{\mathrm{fT}} M_{\tilde{f}_R}^2 V_R^{f*}\right)_{ij} + \hat{M}_{f_i}^2 + D_R^f \end{bmatrix}, \tag{B.6}
$$

where $D_{L,R}^f$ remain diagonal, and $\hat{M}_f$ is the diagonal matrix of the f type fermions. The sources of flavour violation then come from

$$
\left(M^{\mathrm{SCKM}2}_{\tilde{Q}}\right)_{ij} = \left(V_L^{\mathrm{fT}} M_{\tilde{Q}}^2 V_L^{f*}\right)_{ij}
$$

$$
\left(a_f^{\mathrm{SCKM}}\right)_{ij} = - \left(V_L^{\mathrm{fT}} a_f V_R^{f*}\right)_{ij}
$$

$$
\left(M^{\mathrm{SCKM}2}_{\tilde{f}_R}\right)_{ij} = \left(V_R^{\mathrm{fT}} M_{\tilde{f}_R}^2 V_R^{f*}\right)_{ij}, \tag{B.7}
$$

which are not generically diagonal in the SCKM basis. For the case of 1 family the matrix $(\mathcal{M}_{\tilde{f}}^{\mathrm{SCKM}})^2_{ij}$ is the usual 2×2 soft mass matrix whose diagonalization is obtained through

$$
\left[\tilde{f}_L, \tilde{f}_R\right] \begin{bmatrix} M^{\mathrm{SCKM}2}_{\mathrm{LL}} & M^{\mathrm{SCKM}\dagger 2}_{\mathrm{LR}} \\ M^{\mathrm{SCKM}2}_{\mathrm{LR}} & M^{\mathrm{SCKM}2}_{\mathrm{RR}} \end{bmatrix} \begin{bmatrix} \tilde{f}_L^* \\ \tilde{f}_R^* \end{bmatrix} = \left[\tilde{f}_L, \tilde{f}_R\right] K^\dagger \begin{bmatrix} M_{\tilde{f}_1}^2 & \\ & M_{\tilde{f}_2}^2 \end{bmatrix} K \begin{bmatrix} \tilde{f}_L^* \\ \tilde{f}_R^* \end{bmatrix}, \tag{B.8}
$$

where then the sfermion mass eigenstates, $\tilde{q}_i$ are defined by

$$
\begin{bmatrix} \tilde{f}_L \\ \tilde{f}_R \end{bmatrix} = K^T \begin{bmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{bmatrix}. \tag{B.9}
$$

Analogously for three families, we have

$$
\widehat{\mathcal{M}}_{\tilde{f}}^{2\dagger} = \begin{bmatrix} (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{11} & (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{12} & (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{13} \\ (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{12}^\dagger & (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{22} & (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{23} \\ (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{13}^\dagger & (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{23}^\dagger & (\widehat{\mathcal{M}}_{\tilde{f}}^2)_{33} \end{bmatrix}, \quad \tilde{u}' = \begin{bmatrix} \tilde{u}_L \\ \tilde{u}_R \\ \tilde{c}_L \\ \tilde{c}_R \\ \tilde{t}_L \\ \tilde{t}_R \end{bmatrix}, \quad \tilde{d}' = \begin{bmatrix} \tilde{d}_L \\ \tilde{d}_R \\ \tilde{s}_L \\ \tilde{s}_R \\ \tilde{b}_L \\ \tilde{b}_R \end{bmatrix}. \tag{B.10}
$$

Hence the effective soft mass matrix $(\widehat{\mathcal{M}}_{\tilde{f}}^2)_{ij}$ is a 6×6 matrix, and the electroweak eigenstates and mass eigenstates are related through

$$
\tilde{f}' = \left(K^f\right)^T \tilde{f}, \tag{B.11}
$$

f' and f here are column vectors. The primed states are the eigenstates of the Lagrangian in the gauge basis and the unprimed in the mass eigenbasis. Using the above definition of eq. (B.9) and eq. (B.11) the couplings of the mass eigenstates involved in gaugino/ sfermion vertex in terms of the gauge eigenstates are given as follows

$$C_{\tilde{g}\tilde{f}_j^*(f_L)_k} = K_{j(2k-1)}^{f*} C_{\tilde{g}\tilde{f}_{(2k-1)}'^*(f_L')_k}, \quad C_{\tilde{g}\tilde{f}_j^*(f_R)_k} = K_{j(2k)}^{f*} C_{\tilde{g}\tilde{f}_{(2k)}'^*(f_R')_k}. \tag{B.12}$$

We are working in the basis where Y_u is diagonal at the GUT scale. At the EW scale two loop corrections will make it non-diagonal but due to the smallness of the expansion parameter ϵ_u, we are neglecting this contribution. Then $V_{\text{CKM}} = V_L^d$, contrary to the convention used in many other papers (e.g. [14, 19]) where Y_d is taken to be diagonal at the EW scale. Hence the couplings of fermions and sfermions to other supersymmetric particles must be defined accordingly. These couplings are given in the next two appendices.

B.2 Quarks-squarks-neutralino interactions

The interaction between quark-squark and neutralinos it is described by the Lagrangian, in the mass eigenstate basis, by:

$$\begin{aligned} -\mathcal{L}_{q-\tilde{q}-\chi^0} =\ & \tilde{d}_j \bar{d}_k \left[C_{\tilde{\chi}_i^o \tilde{d}_j(d_L^\dagger)_k} \mathcal{P}_L + C_{\tilde{\chi}_i^o \tilde{d}_j(d_R^\dagger)_k} \mathcal{P}_R \right] \chi_i^o \\ & + \tilde{d}_k^\dagger \overline{\chi^o}_i \left[C_{\tilde{\chi}_i^o \tilde{d}_j(d_L^\dagger)_k}^\dagger \mathcal{P}_R + C_{\tilde{\chi}_i^o \tilde{d}_j(d_R^\dagger)_k}^\dagger \mathcal{P}_L \right] d_j, \quad \chi_i^o = \left[\tilde{\chi}_i^o, \overline{\tilde{\chi}_i^o} \right]. \tag{B.13} \end{aligned}$$

The mass Lagrangian is given by $\mathcal{L} = -\frac{1}{2}\tilde{\psi}^0 N N^\dagger \mathcal{M}_{\tilde{\psi}^0} N^* N^T (\tilde{\psi}^0)^T$ + h.c., where the gauge eigenstates are $\tilde{\psi}^o = [-i\tilde{b}, -i\tilde{w}, \tilde{h}_d, \tilde{h}_u]$, for $\tilde{h}_{1,2} = \tilde{h}_{d,u}$. The mass matrix of neutralinos is diagonalized by $M_{\chi^0} = N^\dagger \mathcal{M}_{\tilde{\psi}^0} N^*$, and hence the mass eigenstates, $\tilde{\chi}_i^0$, are given by $\tilde{\chi}_i^0 = (\tilde{\psi}^0 N)_i$.

The couplings of *incoming* neutralinos to sfermion-fermion in the gauge basis, eq. (B.13), can be identified from those of eq. (D20) in [39] as $C_{\tilde{\psi}_i^o \tilde{f}_{(p)}'(f_R')_m} = a_{\tilde{\psi}_i^o \tilde{f}_{(p)}'(f_R')_m}$, and $C_{\tilde{\psi}_i^o \tilde{f}_{(p)}'(f_L')_m} = b_{\tilde{\psi}_i^o \tilde{f}_{(p)}'(f_L')_m}$, for $p = L, R$. Then the coupling at each vertex in the basis of mass eigenstates can be written as $-i(C_{u_R}\mathcal{P}_L + C_{u_L}\mathcal{P}_R)$ and $-i\mathcal{C}^{-1}(C_{d_R}\mathcal{P}_L + C_{d_L}\mathcal{P}_R)$, for up and down quarks respectively. C_{f_R} is any coupling involving a right-handed fermion state, for example $C_{\tilde{\chi}_i^o \tilde{f}_j(f_R)_m}$. Then the couplings of the mass eigenstates involved in neutralino or chargino/ sfermion loops in terms of the gauge eigenstates are given as

$$\mathbf{C}_{ijk}^u = C_{\tilde{\chi}_i^o \tilde{u}_j^\dagger(u_R)_k} = \left[N_{1i}^* K_{j(2k)}^u \left(\frac{g_1}{\sqrt{2}} \right) \mathcal{Y}_{u_R} + N_{4i}^* K_{j(2k-1)}^u \hat{Y}_{kk}^u \right]$$

$$\mathbf{D}_{ijk}^u = C_{\tilde{\chi}_i^{o*} \tilde{u}_j^\dagger(u_L)_k} = \left[N_{1i} K_{j(2k-1)}^u \left(\frac{g_1}{\sqrt{2}} \right) \mathcal{Y}_{u_L} + N_{2i} K_{j(2k-1)}^u (\sqrt{2}g I_3^{u_L}) + N_{4i} K_{j(2k)}^u \hat{Y}_{kk}^u \right]$$

$$\mathbf{C}_{ijk}^d = C_{\tilde{\chi}_i^o \tilde{d}_j(d_R^\dagger)_k} = \left[N_{1i}^* K_{j(2k)}^d \frac{g_1}{\sqrt{2}} \mathcal{Y}_{d_R} + N_{3i}^* K_{j(2k-1)}^d \hat{Y}_{kk}^d \right]$$

$$\mathbf{D}_{ijk}^d = C_{\tilde{\chi}_i^{o*} \tilde{d}_j(d_L^\dagger)_k} = \left[N_{1i} K_{j(2k-1)}^d \frac{g_1}{\sqrt{2}} \mathcal{Y}_{d_L} + N_{2i} K_{j(2k-1)}^d \sqrt{2}g I_3^{d_L} + N_{3i} \hat{Y}_{kk}^d K_{j2k}^d \right] \tag{B.14}$$

B.3 Quarks-squarks-chargino interactions

The quarks-squarks-chargino interactions are given by

$$
\begin{aligned}
-\mathcal{L}_{q-\tilde{q}-\tilde{\psi}^{\pm}} = \ &\tilde{u}_j \bar{d}_k \left[C_{\tilde{\chi}_i^{-c}\tilde{u}_j(d_L^\dagger)_k} \mathcal{P}_L + C_{\tilde{\chi}_i^{+c}\tilde{u}_j(d_R^\dagger)_k} \mathcal{P}_R \right] \chi_i^c \\
&+ \tilde{u}_k^\dagger \overline{\chi^c}_i \left[C^\dagger_{\tilde{\chi}_i^{-c}\tilde{u}_j(d_L^\dagger)_k} \mathcal{P}_R + C^\dagger_{\tilde{\chi}_i^{+c}\tilde{u}_j(d_R^\dagger)_k} \mathcal{P}_L \right] d_i \\
&+ \tilde{d}_j \bar{u}_k \left[C_{\tilde{\chi}_i^{-c}\tilde{d}_j(u_L^\dagger)_k} \mathcal{P}_L + C_{\tilde{\chi}_i^{+c}\tilde{d}_j(u_R^\dagger)_k} \mathcal{P}_R \right] \chi_i^c \\
&+ \tilde{d}_k^\dagger \overline{\chi^c}_i \left[C^\dagger_{\tilde{\chi}_i^{-c}\tilde{d}_j(u_L^\dagger)_k} \mathcal{P}_R + C^\dagger_{\tilde{\chi}_i^{+c}\tilde{d}_j(u_R^\dagger)_k} \mathcal{P}_L \right] u_j, \quad \chi_i = \left[\chi_i^+, \ \overline{\chi_i^-} \right]. \text{(B.15)}
\end{aligned}
$$

The mass Lagrangian is given by $\mathcal{L} = -\frac{1}{2}\left[\tilde{\psi}^+ V V^+ \mathcal{M}_{\tilde{\psi}^+} U^* U^T (\tilde{\psi}^-)^T + \tilde{\psi}^- \mathcal{M}^T_{\tilde{\psi}^+}(\tilde{\psi}^+)^T \right]$, where $\tilde{\psi}^+ = [-iw^+, \ \tilde{h}_u^+]$, $\tilde{\psi}^- = [-iw^-, \ \tilde{h}_d^-]$, $\tilde{h}_{1,2} = \tilde{h}_{d,u}$. The mass matrix is diagonalized by $M_{\tilde{\chi}^\pm} = V^+ \mathcal{M}_{\tilde{\psi}^+} U^* = U^+ \mathcal{M}^T_{\tilde{\psi}^+} V^*$, hence the rotation to chargino mass eigenstates is $\tilde{\chi}^+ = (\tilde{\psi}_1^+, \ \tilde{\psi}_2^+)V$, $\tilde{\chi}^- = (\tilde{\psi}_1^-, \ \tilde{\psi}_2^-)U$.

The couplings of *incoming* charginos to sfermion-fermion in the gauge basis can also be identified to those of eq. (20) of [39]. Using the transformations described in this section to go to the eigenmass basis we have

$$
\begin{aligned}
\mathbf{A}_{ijk}^u &= C_{\tilde{\chi}_i^+ \tilde{d}_j^*(u_L)_k} = V_{2i}^* \sum_{n=1}^3 \left[K_{j(2n-1)}^{d*} \left(V_{\mathrm{CKM}}^\dagger \right)_{nk} \left(-\hat{Y}_{kk}^u \right) \right] \\
\mathbf{B}_{ijk}^u &= C_{\tilde{\chi}_i^{-c}\tilde{d}_j^*(u_R)_k} = \sum_{n=1}^3 \left[U_{1i} K_{j(2n-1)}^{d*} \left(V_{\mathrm{CKM}}^\dagger \right)_{nk} g \ + U_{2i} K_{j(2n)}^{d*} \left(-\hat{Y}_{nn}^d \right) \left(V_{\mathrm{CKM}}^\dagger \right)_{nk} \right] \\
\mathbf{B}_{ijk}^d &= C_{\tilde{\chi}_i^{+c}\tilde{u}_j(d_R^\dagger)_k} = \sum_{n=1}^3 \left[V_{2i} K_{j(2n)}^u \left(V_{\mathrm{CKM}}^\dagger \right)_{kn} \left(-\hat{Y}_{nn}^u \right) + g V_{1i} K_{j(2n-1)}^u \left(V_{\mathrm{CKM}}^\dagger \right)_{kn} \right] \\
\mathbf{A}_{ijk}^d &= C_{\tilde{\chi}_i^- \tilde{u}_j(d_L^\dagger)_k} = U_{2i}^* \sum_{n=1}^3 \left[K_{j(2n-1)}^u \left(-\hat{Y}_{kk}^d \right) \left(V_{\mathrm{CKM}}^\dagger \right)_{kn} \right],
\end{aligned} \tag{B.16}
$$

where it is useful to remember than in this notation the components $K_{j(2n-1)}^f$ are related to left-handed transformations and K_{j2n}^f with right-handed ones.

Acknowledgments

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